

William of Ockham's Dialectic of Science --

A Dialectical Episode in Early Medieval Philosophy

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Author's Preface. The purpose of F.E.D. Vignette #13 is to present an E.D. 'Dialectical Model Equation' for a systematic dialectical method of presentation of the Early Medieval Philosophy of Science of William of Ockham.

This 'Dialectical Model Equation' also serves as a "worked", "cook-book" example, and sample, of the application of the NQ dialectical algebra to help organize, and to compactly encode, systematic expositions for "'synchronic snapshots'" of the structure/process of the living self-sustenance of systems -- 'physio-systems', 'ideo-systems', and hybrids of the two, such as the system of Science -- that are presently in existence; expositions that explicate and assess the present content of such systems, without explicitly expositing their "'diachronic chronicles'" -- neither explicitly reconstructing the prior, diachronic history that constituted that content, nor explicitly 'pre-constructing' any presently-predicted future, successor system of such. [Note: the ordering Mythopoeia, Religion, Philosophy, Science, Psychohistory is, per the E.D. first Psychohistorical-Dialectical 'Meta-Equation', that of Human Ideology/Knowledge 'Meta-Evolution'. See http://www.dialectics.org/dialectics/Aoristos_Blog/Entries/2012/5/19_The_F.E.D._Psychohistorical_Equations.html].

A Note about the On-Line Availability of Definitions of F.E.D. Key Technical Terms. Definitions of Encyclopedia Dialectica technical terms, including of E.D. 'neologia', are available on-line via the following URLs --

<http://www.dialectics.org/dialectics/Glossary.html>

<https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/ClarificationsArchive.htm>

-- by clicking on the links associated with each such term, listed, in alphabetic order, on the web-pages linked-to above.

Definitions of the Encyclopedia Dialectica special terms most fundamental to this vignette are linked-to below --

«arché»

<https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Arche/Arche.htm>

«arithmos» and «arithmoi»

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Arithmos/Arithmos.htm>

«aufheben»

<https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Aufheben/Aufheben.htm>

Diachronic vs. Synchronic

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Synchronic/Synchronic.htm>

NQ dialectical arithmetic/algebra

http://www.dialectics.org/dialectics/Correspondence_files/Letter17-06JUN2009.pdf

Systematic or Synchronic Dialectics

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/SystematicDialectics/SystematicDialectics.htm>

-- and we plan to expand these definitions resources as the Encyclopedia Dialectica Dictionary Project unfolds.

[Note: "'Arithmetical Quantifiers'" vs. 'Arithmetical Qualifiers'. In the phrase "3 apples", we term "3" the "arithmetical ["pure"-]quantifier", and "apples" the "ontological" -- or kind of thing -- "'qualifier'". In the phrase "3 pounds of apples", we term "pounds" the 'metrical[-unit] qualifier' -- or "'unit of measure qualifier'" -- quantified by the 3, which, together, 'quanto-qualify' the 'ontological qualifier', "apples". A key use-value of the dialectical arithmetics is to provide algorithmic, ideographical-symbolic systems for the various kinds of 'arithmetical qualifiers', both with and without the co-presence of "'arithmetical quantifiers'".]

I. Introduction to the Subject *Systematic Dialectic* within *Medieval Occidental Philosophy*. The present **F.E.D.** vignette sets forth a *dialectical-ideographical model* of William of Ockham's circa **1323** C.E. *philosophical theory* /- 'ideo-taxonomy' of *the sciences*. Ockham lived circa **1290** to **1349** C.E. [Common Era].

This vignette contains this example of *dialectical-mathematical models* in *general*, formulated using a *specific dialectical-mathematical language* -- that of the "purely-*qualitative*", "purely connotative" mathematics of the '*algorithmic-heuristic algebra*' of the **NQ** '*arithmetic of dialectics*', which is an *axiomatized algebraic dialectical logic*.

For further background on the algorithmics and the axiomatics of its *ideographical*, or "*symbolic*" *dialectical logic* -- or '*algebraic logic of dialectics*' -- see the www.dialectics.org website's [Briefs Page](#) as well as section **II.** of this vignette.

The core of Ockham's *philosophical theory of the sciences* is as follows.

Ockham gave to terms of language that refer to 'non-term' "*physical facts*", the descriptor [the term] "*categorematic*".

Thus, the "term", or word, "rock" refers to that "external-to-mind **r**eality", and is therefore a "*categorematic*" term.

Ockham gave to **t**erms of language that refer to, e.g., "*categorematic*" terms, i.e., to other terms of language itself, the descriptor [the term] "*syncategorematic*".

Thus, the "terms", or words, "all", "some", "none", and "not" are all examples of such "word-referring words", or "term-referring-**t**erms".

Ockham also labeled terms that refer to things belonging to the **r**eality outside of [the reality of] language, by the phrase "*terms of the **f**irst intention*".

He labeled **t**erms that refer to *terms of the **f**irst intension* by the phrase "*terms of the **s**econd intention*".

In the most direct sense, then, "*syncategorematic*" **t**erms are "*terms of the **s**econd intention*".

However, Ockham distinguishes *mere* "terms" of language, in the sense of *individual words*, from those higher-level /- higher 'qualo-fractal scale' language constructions [constructions that we of **F.E.D.** call '*meta-terms*', or '*meta-words*', *each one made up out of a* [usually] *heterogeneous multiplicity of mere* "terms", of *mere* individual words] which are named "*complete sentences*", or "*well-formed propositions*".

Ockham does not "*reduce*" the latter '*ideo-ontology*' to the former.

Thus, for Ockham, *categorematic* **t**erms, in 'sentence-ial' *combination* with *syncategorematic* **t**erms, in the form of well-formed *sentences* or *propositions*, and, thus, *functioning together*, refer to, and describe, *more adequately*, typically, the things belonging to the **r**eality outside of human language, than do single, isolated, "*mere*" **t**erms, or single, isolated words, such as those **t**erms/words which are named "names", or "nouns".

The *sentence/proposition* -- "*All natural objects are transitory.*" -- is thus *first intentional*. Note: this sentence "contains" [*combines*] both *categorematic* **t**erms, like "natural objects", and also *syncategorematic* **t**erms, like "*All*".

The *sentence/proposition* -- "*All second-order-formal-logic axiomatizations of "Natural Numbers" arithmetic are either formally inconsistent or formally incomplete.*" -- is thus *second intentional* [at least!]. Note that this sentence "contains" no **t**erms which refer to *non*-linguistic, "*physical*" *objects*, but only **t**erms which refer to other **t**erms, or to the names of "*mental objects*" / "*idea-objects*", to 'meme-ic' or 'memetic', human-Phenomic, '*psychohistorical materialities*'.

On the basis of these distinctions, Ockham divides the «*Genos*» of the "*Sciences*" into two «*species*», or '*sub*¹-«*Gene*» --

«*species*» **1**: «*Scientia realis*», encompassing the *sciences* of "*real* things" [i.e., of *external*-to-mind, physical things]. For example, the science of *biology* would form a *sub*¹-«*species*» of this «*species*». This «*species*» is *first intentional*.

«*species*» **2**: «*Scientia rationalis*», encompassing the *sciences* of '*noetic*-things' [i.e., of *internal*-to-mind, 'Phenomic' things]. For example, *mathematics* would form a *sub*¹-«*species*» of this «*species*». This «*species*» is *second intentional*.

For further background on Ockham's theory, see W. L. Reese, *Dictionary of Philosophy and Religion, Eastern and Western Thought*, Humanities Press [Atlantic Heights, NJ: 1980], pp. 627 ff.

The two passages from the source cited above that are most relevant to this vignette are extracted below:

"...Ockham makes an important distinction between *categorematic* and *syncategorematic* terms [A.D.: a distinction in logic which harks back at least to the Stoics; cf. also the work of Ockham's predecessor, William of Sherwood, circa 1210 to 1270 C.E.]."

"Terms which refer to reality are *categorematic*, while terms which refer to syncategorematic terms are *syncategorematic*."

"Most common nouns are *categorematic*, while words such as "not," "all," and "some" are *syncategorematic*."

"...Terms that refer to [A.D.: external-to-language/-mind] things are called terms of *first intention*."

"Terms referring to terms of first intention are called terms of *second intention*."

"In one sense the syncategorematic terms mentioned above are second intentional; but, as Ockham wishes to use the distinction, categorematic terms and syncategorematic terms, functioning together, refer to [A.D.: external-to-language/-mind] things. "All men are mortal" is thus first intentional."

"But when we use terms such as "genus," "species," and "difference" we are using terms of second intention."

"Propositions utilizing such terms refer not to the world [A.D.: i.e., to the physical world, external to the human-language/-mind "world"] but to terms of first intention."

"...On the basis of the distinction just mentioned, Ockham divides the sciences into two types [A.D.: into two «*species*»].

"*Scientia rationalis*, or rational science, is second intentional. Logic is a science of this type."

"*Scientia realis* is first intentional. Physics is an example of a science of real [A.D.: external-to-language/-mind] things."

The same source gives, elsewhere [p. 563], a useful excursion into the 'philosophical etymology' of the term "syncategorematic":

"From the Greek *syn* ("together") and *categorema* ("predicate")."

"The derivation may refer to those terms which go together with the predicates or [to] those terms which hold the predicates together; and thus, [to] those terms which cannot be [intelligibly] used by themselves but only in conjunction with other terms: e.g., "all," taken alone, has no definite reference to anything, but "all men" has.

" "All" is syncategorematic and "men" is categorematic."

"In addition to the quantifiers "all," "some," and "none," included among syncategorematic terms are "not," "if...then," "either...or," and "both...and."

II. E.D. Standard Interpretations for the Initial Generic Ordinal Qualifiers of the $\mathbf{N}^{\mathbf{Q}}$ Arithmetic. Perhaps a bit surprisingly, upon first apprehension, the first four, first-order-logic, Dedekind-Peano Postulates for the “Natural” Numbers focus on their *ordinality*, not on their *cardinality*, viz. --

1. **1** is a “Natural Number”.
2. The *successor* of any “Natural Number” is also a “Natural Number”.
3. No two, distinct “Natural Numbers” have the same *successor*.
4. **1** is not the *successor* of any “Natural Number”, i.e., **1** has no ancestor within the “Natural Numbers”.

These postulates thus identify the essence of the “Natural Numbers”, explicitly, in terms of [apparently purely-] *quantitative ordinality*. In keeping with this focus on *the ordinal*, Seldon defines the system of the $\mathbf{N}^{\mathbf{Q}}$ *dialectical arithmetic* -- the first ‘*antithesis-system*’, or ‘*contra-system*’, to the “Natural Numbers” as «*arché*»-*system* -- in terms of *qualitative ordinality*. The $\mathbf{N}^{\mathbf{Q}}$, which he also calls the ‘*meta-Natural meta-Numbers*’, are, in their simplest, least-interpreted essence, a consecutive sequence of ‘meta-numeral’ ideograms representing the successive *qualities*, not the *quantities*, of *ordinality* -- the *quality* of ‘*first-ness*’, followed by the *quality* of ‘*second-ness*’, followed by the *quality* of ‘*third-ness*’, and so on... -- satisfying the four first-order-logic ‘*contra-Peanic*’, ‘*Qualo-Peanic*’ axioms:

- 1’. The *ordinal qualifier* for the *quality* of ‘*first-ness*’ is an element of the ‘*consecuum*’ of generic *ordinal qualifiers*.
- 2’. The *successor* of any element of the ‘*consecuum*’ of generic *ordinal qualifiers* is also an element of same.
- 3’. Any two, distinct *ordinal qualifiers* have *qualitatively unequal successors*.
- 4’. The *ordinal qualifier* for the *quality* of ‘*first-ness*’ is «*arché*»: not the *successor* of any element of its ‘*consecuum*’.

The symbols, or ‘*meta-numerals*’, that stand for the ‘*meta-numbers*’ of the $\mathbf{N}^{\mathbf{Q}}$ ‘*archeonic consecuum*’ are derived, *syntactically*, in a way which represents the *semantic self-subsumption*, ‘*self-subordination*’, or ‘*self-demotion*’ [*dialectical, self-«aufheben» self-negation*] of the “Natural Numbers”. That derivation is part of the positive fruition of the *dialectical, immanent self-critique* of the “Natural Numbers”, which divulges the $\mathbf{N}^{\mathbf{Q}}$ as the implicit, most extreme known *opposite*, “*Non-Standard Model*” of the “Standard”, Peano “Natural Numbers”. It involves the turning of the ‘*generic ordinal quantifiers*’ of the “Natural Numbers” into the ‘*generic ordinal qualifiers*’ of the $\mathbf{N}^{\mathbf{Q}}$ ‘*meta-Natural meta-Numbers*’. The conceptual ‘*self-subsumption*’ of the *quantitative ordinality* intended by the Dedekind-Peano Postulates surfaces the $\mathbf{N}^{\mathbf{Q}}$ as their hitherto hidden, implicit ‘*intra-dual*’, based upon the *generic quality of ordinality*, a “‘*genericity*’” which we represent by the ‘*meta-numeralic*’ ideogram ‘**q**’. That ‘*meta-numeral component*’ represents ‘*qualitative ordinality*’, or ‘*ordinal quality*’, in general: just ‘**q**’, or, more fully expressed, just \mathbf{q}_N .

To fully express, ‘*meta-numeral-y*’, or ideographically, the ‘*consecuum*’ of *specific ordinal qualities*, namely --

$$\mathbf{N}^{\mathbf{Q}} \equiv \{ \textit{‘first-ness’}; \textit{‘second-ness’}; \textit{‘third-ness’}, \text{etc.} \}$$

-- we must add a second ‘*meta-numeral component*’, via “‘*subordinating*’” *specific* “Natural Numbers”, as *specific ordinal quantifiers*, to the *generic ordinal qualifier* symbol ‘**q**’, by ‘*subscripting*’ those *specific* “Natural Numbers” to a ‘*script-level*’ ‘**q**’, “‘above’” them, yielding --

$$\mathbf{N}^{\mathbf{Q}} \equiv \{ \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \dots \} \text{ [in which each ‘meta-number’ is a minimal, «genos»/«species» «arithmos eidetikos» in its own right],}$$

vs.

$$\mathbf{N} \equiv \{ 1, 2, 3, \dots \}.$$

Note that this *opposition* of an arithmetical system of *purely-quantitative ordinality*, based upon the **N**, *versus* an arithmetical system of *purely-qualitative ordinality*, based on the **N_Q**, is not a *radical dualism*, imagined as an *absolute, irreconcilable diremption* between an *absolute quantitative* and an *absolute qualitative*. This *opposition* is, on the contrary, a *dialectical antithesis-sum*. The **N** *quantifiers* are still there, as *specifiers* -- still present -- in, or “‘under’”, the generic **qs** of the **N_Q** *qualifiers*, though *subsumed*, *subordinated*, *demoted* -- demoted to being their mere *subscripts* or *denominators*: The **N** *quantifiers* are still “‘contained” in[side] [“‘beneath”] the **N_Q** *qualifiers*. That is, each **N_Q** *qualifier* is an «*aufheben*» *determinate negation / conservation / elevation-into-one-step-higher-generality* of an individual **N** *quantifier*.

The joint «*aufheben*» *elevation* of the *quantitative ordinality* «*species*» yields the «*genos*» ‘**q**’, of the *qualitative ordinality*.

For this first layer of interpretation of these “‘purely-qualitative” **N_Q** ‘*meta-numbers*’ -- which does not yet make explicit their universal interpretability for the modeling of *dialectical progressions* -- this is all that they represent: *abstract temporality*; [*abstract chronological*] *order*; *generic ‘ordered-ness’*; the consecutive succession of ‘*qualitative ordinality*’; the ‘*consecuum*’ of *order quality* or of *order qualities*.

But even here, at this minimally-interpreted stage of the construction of the **N_Q** *dialectical arithmetic*, there is already a kind of *generic ‘connotative entailment*’ at work. True, it is but a shadow, and but a ‘pre-vestigial’ harbinger, of the richness of the kind of particularity of *categorial followership* that drives forward, intuitively, the *dialectical, purely-qualitative logic* of the more concrete, more *specific dialectical-algebraic* interpretations thereof. A case in point is exemplified in the very **N_Q** *algebraic model* Ockham’s *dialectic of Science* constructed herein.

This *generic ‘connotative entailment*’ can be formulated as follows: ‘*second-ness*’ follows -- and even follows *from* -- ‘*first-ness*’; ‘*third-ness*’ follows [*from*] ‘*second-ness*’, and so on.

In the next section, the construction, by iterated interpretations layering, of the Seldonian *first dialectical arithmetic* will advance from this harbinger of ‘*connotative entailment*’ to the following, still *generic*, but at last also explicitly *dialectical*, form of ‘*connotative entailment*’ *ordinality*: ‘*first full antithesis*’ follows from the *self-interaction* of [*«arché»-thesis*]; *first full synthesis* follows from the *mutual interaction* of *first full thesis* and *first full antithesis*, and so on.

For example, if we take epoch $\mathbf{s} = 1$, and denote the «arché» *ontological category* simply by $\underline{\mathbf{a}}$, for syntactical convenience, then the *Dyadic Seldon Function* calls for the following, as per the \mathbf{nQ} axioms, since $2^1 = 2$ --

$$\underline{\mathbf{a}} \underline{\mathbf{a}}_1 = (\underline{\mathbf{a}})^{2^1} = (\underline{\mathbf{a}})^2 = (\underline{\mathbf{a}}) \otimes (\underline{\mathbf{a}}) = \underline{\mathbf{a}} \oplus \underline{\mathbf{b}}$$

-- wherein $\underline{\mathbf{a}} [\text{---}] \mathbb{Q}_1$ connotes the «arché» *category* or “‘*thesis*’”, and where $\underline{\mathbf{b}} [\text{---}] \mathbb{Q}_2$ connotes the *first ‘contra-category’* or “‘*antithesis*’”, with ‘ \oplus ’ standing for a *generalized addition* operation, that encompasses the *addition* of *qualitatively* distinct terms, and with ‘ \otimes ’ standing for a *generalized multiplication* operation, that encompasses the *multiplication* operation *specific* to the \mathbf{nQ} *qualifiers*.

If we take ‘ $(\underline{\mathbf{a}}) \otimes (\underline{\mathbf{a}})$ ’ with $(\underline{\mathbf{a}})$ connoting the *category to be dialectically negated*, and with ‘ $(\underline{\mathbf{a}})$ ’ connoting the *category doing that dialectically [self-]negating*, with ‘ $(\underline{\mathbf{a}})$ ’ as the *dialectical, determinate ‘negation-sign’ specific* to $(\underline{\mathbf{a}})$ as the object of the so-indicated *dialectical, determinate negation operation*, then we have ‘ $(\underline{\mathbf{a}}) \otimes (\underline{\mathbf{a}})$ ’, as a whole, as connoting the *first dialectical negation, by the negation-operation $(\underline{\mathbf{a}})$ of the self-same negation operation, $(\underline{\mathbf{a}})$, that is also the operand of the negation-operator $(\underline{\mathbf{a}})$* , forming what Seldon calls an ideographical ‘subject [$(\underline{\mathbf{a}})$]-verb [$(\underline{\mathbf{a}})$]-object [$(\underline{\mathbf{a}})$] identical’, and yielding the *first dyad*: “‘*thesis* \oplus *antithesis*’”, or ‘«arché» *category* \oplus *first contra-category*’, viz. --

$$(\underline{\mathbf{a}}) \otimes (\underline{\mathbf{a}}) = \mathbf{q}_a \otimes \mathbf{q}_{aa} = \mathbf{q}_a \otimes \mathbf{q}_b \equiv \underline{\mathbf{a}} \oplus \underline{\mathbf{b}}$$

-- which, in terms of the *generic*, minimally-interpreted \mathbf{nQ} *arithmetic*, is a *dialectical interpretation* of the *generic* --

$$\underline{\mathbf{a}} \underline{\mathbf{a}}_1 = \llbracket \mathbf{q}_1 \rrbracket^{2^1} = \llbracket \mathbf{q}_1 \rrbracket^2 = \llbracket \mathbf{q}_1 \rrbracket \otimes \llbracket \mathbf{q}_1 \rrbracket = \llbracket \mathbf{q}_1 \oplus \mathbf{q}_{1+1} \rrbracket = \mathbf{q}_1 \oplus \mathbf{q}_2$$

The *second* iteration of this *dialectical negation of the negation*, corresponding to the consecutively *next* value of \mathbf{s} namely, $\mathbf{s} = 2$, for the \mathbf{nQ} *arithmetic* interpreted for [*psycho*]*historical dialectics*, yields the following, ontologically-expanded *cumulum* of *ontological categories* -- a ‘*dyad of dyads*’, consisting of 4 consecutive *ontological categories*:

$$\underline{\mathbf{a}} \underline{\mathbf{a}}_2 = (\underline{\mathbf{a}})^{2^2} = (\underline{\mathbf{a}})^4 = ((\underline{\mathbf{a}})^2)^2 = (\underline{\mathbf{a}} \oplus \underline{\mathbf{b}})^2$$

$$= (\underline{\mathbf{a}} \oplus \underline{\mathbf{b}}) \otimes (\underline{\mathbf{a}} \oplus \underline{\mathbf{b}})$$

$$= \underline{\mathbf{a}} \oplus \underline{\mathbf{b}} \oplus \underline{\mathbf{c}} \oplus \underline{\mathbf{d}}$$

The additional **2** ontological category-symbols above are *dialectically interpreted*, per the E.D. standard, as follows:

c = *third ontological category, first full uni-category*;

d = *fourth ontological category, second contra-category*.

We will not here pursue this E.D. standard *dialectical interpretation* of the *ontological categories* generated by the generic *Dyadic Seldon Function* beyond $\tau = 2$, because the “‘solution’” -- or ‘*semantification*’ -- of the category-terms generated by the ‘*dialectical-mathematical meta-model meta-equation*’ constructed in this vignette [whose terms are generated, initially, as *algebraic unknowns*, terms of *unknown* meaning], as presented herein, does not extend beyond that *second ‘self-iteration’* for that ‘*meta-model*’.

The ‘*purely-qualitative calculations*’ illustrated above describe our expectations for this ‘*meta-model*’ in terms of *generic* characterizations of the successive, consecutive *dialectical categories*.

The next section addresses the heart of this ‘*meta-model*’ -- the *specific* meanings of the *generic dialectical categories* as applied to the *special case* of *the systematic dialectic of the Early-Medieval Philosophy of Science* of William of Ockham.

IV. E.D. Solution for the 'Meta-Model Meta-Equation' of Ockham's Dialectic of Science. The core content of this vignette is a recapitulation, and an immanent extension, of our 16 June 2009 exposition of this topic, available via the [Correspondence Page](#) of the www.dialectics.org website: http://www.dialectics.org/dialectics/Correspondence_files/Letter18-16JUN2009.pdf.

To begin our model of Ockham's *dialectical taxonomy* of this universe-of-discourse of human ideas, we use the *dialectical logic* axiomatized in an earlier letter available via the [www.dialectics.org Correspondence Page](#) -- http://www.dialectics.org/dialectics/Correspondence_files/Letter17-06JUN2009.pdf

-- and we enact the “‘mental action’” / “‘mental operation’” signified by the ideogram ‘(—)’. That is, we “interpret” or “assign” q_1 , the *first* of the *generic* ‘meta-Natural meta-Numbers’ of our NQ , ‘unquantifiable Qualifier’ ‘*dialectical ideography*’, or ‘*dialectical symbolic logic*’ [‘*dialectical-ideographic logic*’], as follows --

$$q_1 (—) q_r \text{ or } q_1 (—) r$$

-- such that $r (—) r$ “reality”, the universe of “real things”, outside of human language, outside of the human mind.

Our ‘*dialectical-ideographic model*’ then captures a sequential, systematically-ordered, “‘ordinal-ized’” *presentation* of Ockham’s *theory of the sciences*, using the connotations of the series of ideographical “shorthand” symbols that it *generates* algorithmically.

It does so via r as the *recurrently* ‘self-operating operator’, ‘self-operation’, ‘self-function’, ‘self-refluxive function’, “‘self-reflexive function’”, or *recurrently* self-applied, connotationally specific, determinate *dialectical*, «*aufheben*»-*negation operation* at the heart of the ‘*self-iteration*’ formula --

$$\underbrace{) _ | _ (}_{s \uparrow} = (r)^{2^{s \uparrow}}$$

-- wherein s , as the higher exponent of the exponent 2 , counts the “*s*steps” or “*s*stages” of that *systematic presentation* of the ‘*idea-ontology*’, or ‘*ideo-ontology*’-- of the ‘*ideo-taxonomy*’ -- of the *philosophical categories*, and wherein $\underbrace{) _ | _ (}_s$ denotes the ‘*cumulum*’ -- meaning the connotative and non-amalgamative, “‘*qualitative sum*’”, or “‘*superposition*’” -- of the *categories* presented / ‘*acCumulated*’ as of *s*step s of this “shorthand” “‘*systematic-dialectical*’” ‘*present-ation*’.

Let us, then, unfold this model *s*step-by-*s*step, starting with *s*step $s = 0$, then moving on to *s*step $s = 1$, and thence onward to *s*step $s = 2$, and, finally, to *s*step $s = 3$, a *s*step which *immanently extends* Ockham’s theory to a *category* of *The Sciences* that Ockham, as far as we know, did not envision.

Step 0. The 0th *s*step of this ‘*presentation-model*’, i.e., for $s = 0$, since $2^0 = 2^{+1}/2^{+1} = 2^{+1} \times 2^{-1} = 2^{+1-1} = 1$, yields --

$$\underbrace{) _ | _ (}_0 = (r)^{2^0} = r^1 = r$$

-- which simply reiterates the starting point -- the «*arché*» ‘*ideo-taxonomic category*’ -- of this ‘*present-ation*’.

¿Does this categorial-singleton of *s*step 0 grow into a ‘*categorial-cumulum*’ in *s*step 1?

Step 1. For *s*step $s = 1$ of this ‘*presentation-model*’, since $2^1 = 2^{+1} = 2$, we have --

$$\underbrace{) _ | _ (}_1 = (r)^{2^1} = r^2 = r(r) = r \text{ “of” } r = r \text{ “squared”}$$

-- which connotes the ‘*self-reflexive function*’, or ‘*self-operation*’, of the *dialectical* [«*aufheben*»] *operation* denoted by r .

Syntactically, per the axioms of the \mathbf{nQ} ‘meta-numbers’, we have the following values as the content of step 1:

$$\underline{\mathbf{r}}_1 = (\mathbf{r})^2 = \mathbf{r}^2 = \mathbf{q}_r = \mathbf{q}_r \oplus \mathbf{q}_{rr} \quad [\text{---}] \quad \mathbf{q}_1 \vdash \mathbf{q}_{1+1} = \mathbf{q}_1 \vdash \mathbf{q}_2.$$

Our challenge, for this step, is to “solve for” the new category, \mathbf{q}_{rr} , that is, to determine the meaning, the connotation, of that new ‘categorogram’, given the already known/assigned meaning, or connotation, of the ‘categorogram’ $\mathbf{r} \equiv \mathbf{q}_r$.

So, let us apply the general “‘canon of interpretation’” of such dialectical categories to the specific case of this example.

Per that “‘canon’”, the «*arché*» ‘categorogram’, \mathbf{r} , is to be interpreted, as stipulated, as connoting an «*arithmos*» of «*monads*» -- an assemblage of units -- each one of which is a “*fact*”, a physical, empirical “rreality”, “outside of” language and of the human mind. Also per that “‘canon’”, a symbol like \mathbf{q}_{rr} connotes the new category/«*arithmos*» that results from the self-«aufheben» of the \mathbf{q}_r category/«*arithmos*», and, quite often, that results from the self-«aufheben» ‘meta-monadization’ of some of the «*monads*» of the \mathbf{q}_r category/«*arithmos*».

That is, each «*monad*» or unit of the \mathbf{q}_{rr} «*arithmos*» / category / assemblage of units should be a ‘meta-«monad»’, made up out of a multiplicity -- made up out of a ‘sub-«*arithmos*»’ -- of the «*monads*» of the \mathbf{q}_r «*arithmos*» / category.

So, the units of the \mathbf{q}_{rr} assemblage should be, in this specific case, ‘meta-rrealities’, ‘realities of second degree’, each one made up out of a heterogeneous multiplicity of ‘rrealities of first degree’.

Also, the category \mathbf{q}_{rr} should be another category -- other than and to the physical/empirical “rrealities” category; a ‘meta-category’ to, and a ‘contra-category’ of, the \mathbf{r} category. Moreover, the \mathbf{q}_{rr} symbol should point to a category that is already known to us, that is part of our synchronic, present conceptualized experience of the totality of The Sciences, and to a category that is ontologically different than, and that is richer in determinations than -- that is “more determinate” than -- that is more “complex” and more ‘thought-concrete’ than, the \mathbf{r} category.

On the bases set forth above, we propose that our solution -- our ‘semantification’ -- of the syntactic term \mathbf{q}_{rr} should be \mathbf{t} , connoting the «*arithmos*»/category of terms of human language: $\mathbf{q}_{rr} = \mathbf{q}_t = \mathbf{t} [\text{---}] \mathbf{q}_2$.

We propose this solution, in part, because the terms of human language, as units, are meta-units of the units of empirical/physical “rreality”. Each term of human language refers to, and represents, a heterogeneous multiplicity of similar physical “*facts*”. The term “apple” maps to a large, heterogeneous multiplicity of similar, physical apples. The term “orange” maps to a large, heterogeneous multiplicity of similar, physical oranges. That is, “terms” are general relative to physical “*facts*”. One “term” typically stands for a large, although still finite, set of similar physical situations. Each “term” is an «*aufheben*» ‘meta-«monad»’ of a large number of similar “*fact*”-«*monads*». Our solution for \mathbf{q}_{rr} is that \mathbf{q}_{rr} connotes the «*arithmos*»/category which has terms of human language as its «*monads*».

In an historical, diachronic sense, \mathbf{r} “*of*” \mathbf{r} also connotes the natural-historical result of the ‘self-reflexion’, of the ‘bending-back-upon-self’, of “rreality”; of the turning back, upon itself, of physical “rreality”, to form physical objects -- new parts of that physical “rreality” -- that are capable of conscious reflection upon physical “rrealities”, including upon themselves. In that historical, diachronic context, this would mean the immanent emergence, within pre-human[oid] Nature, from out of pre-human[oid] Nature -- as part of the dialectic, or of the self-development, of Nature itself -- of “observing eyes”, and of “observing sensoria” in general; of observing and thinking minds, as also “things” / objects / bodies within Nature -- i.e., the emergence of natural ‘[ev]entities’ which produce languages, hence “terms”, to name / communicate their observations / experiences among themselves, as a crucial part of their “struggle for existence” -- of their struggle for successful expanded human[oid]-social reproduction.

But this consideration does not belong to *synchronic, systematic dialectics* in itself.

It only represents the *diachronic* implications -- the pointers to *past history*, and to *future history* -- that are *implicit* in *synchronic dialectics*, but which do *not*, as such -- as *diachronic* -- form an *explicit* component of the explanatory apparatus of *synchronic, systematic dialectics*.

Marx's treatises on the *dialectical, immanent critique* of the capitalist, ideology-ridden *science* of "political economy", are works of *synchronic, systematic dialectics*. But, in Marx's view, *systematic dialectics* is *not* absolutely dirempt from *diachronic, historical dialectics*.

Marx discussed this *interconnexion* between *systematic dialectics* and *historical dialectics*, as it arises in his extension of previous concepts of *dialectics* -- an *interconnexion* which is, e.g., inaccessible to the *eternally systematic, Parmenidean dialectics* of Plato's *immutable «arithmos eidetikos»* -- in a crucial methodological passage from the *Grundrisse*:

"...our [A.D.: *systematic-dialectical*] method indicates the points where *historical* investigation must enter in, or where bourgeois economy as a merely *historical* form of the production process *points beyond itself to earlier historical* modes of production."

"In order to develop the laws of bourgeois economy, therefore, it is *not* necessary to write the *real history of the relations of production*."

"But the correct observation and deduction of these laws, *as having themselves become in history*, always leads to primary equations -- like the empirical numbers e.g. in natural science -- which *point towards a past* lying *behind* this system."

"These *indications* [Adeutung], together with a correct grasp of the *present*, then also offer the key to the understanding of the *past* -- a work in its own right which, it is hoped, we shall be able to undertake as well."

"This correct view likewise leads at the same time to points at which the *suspension* [at which the «aufheben» -- A.D.] of the *present* form of production relations gives signs of its *becoming -- foreshadowings of the future*."

"Just as, on one side the pre-bourgeois phases appear as *merely historical*, i.e. *suspended* [i.e., «aufheben»-ed' -- A.D.] pre-suppositions, so do the *contemporary* conditions of production likewise appear as engaged in *suspending themselves* [i.e., in «aufheben»-ating themselves' -- A.D.] and hence in positing the *historic presuppositions* for a new state of society."

[Karl Marx, *Grundrisse: Foundations of the Critique of Political Economy (Rough Draft)*, M. Nicolaus, transl., Penguin Books [Middlesex: 1973], pp. 460-461, italics only emphasis by Marx; *combinations* of italic, underlined, **bold**, and color emphasis have been added by A.D.].

Accepting this solution, our *step s = 1 dialectical-model equation* thus becomes [with '∩' for the *systematic-dialectic* context's 'curvilinear, ontological *delta* operator', signifying the addition of an increment of new 'ideo-ontology'] --

$$\underline{\text{I}}-\underline{\text{I}}-(\underline{\text{r}})^2 = (\underline{\text{r}})^2 = \underline{\text{r}}^2 = \underline{\text{q}}_r^2 = \underline{\text{r}} \oplus \underline{\cap} \underline{\text{r}} = \underline{\text{r}} \oplus \underline{\text{t}} \quad [\text{---}) \underline{\text{q}}_1 \text{I} \underline{\text{q}}_2]$$

-- and we have the meaning that, as of *step 1, s = 1*, our model, $\underline{\text{I}}-\underline{\text{I}}-(\underline{\text{r}})^2$, posits the *possible* existence

of *two* philosophical categories -- **(1)** the category of physical "*r*realities", "'plus'" [signed by '⊕', for connotatively "interpreted", or "assigned", *ontological qualifiers*, in the 'curvilinear symbols motif' context of *systematic dialectics*], **(2)** its 'first contra-category', of "'terms of language'", *both* together comprising **(1)** the «*physis*»-«*monads*», or *fact-units*, of the *physical r*realities' «*arithmos aisthetos*», or sensorially-perceived "*facts*"-*assemblage*, superposed with, and "'*named*'" -- in a *many-to-one* relationship -- *by* **(2)** the 'trans-Platonian' «*arithmos eidetikos*» of *t*terms-units, or of *t*terms-«*monads*».

¿How does this *two*-dimensional 'possibility-space', or categorial 'ideo-ontology cumulum', expand in *step 2*?

Step 2. For step $s = 2$ of this presentation-model, since $2^2 = 2 \times 2 = 4$, we have --

$$\underline{\underline{)}}\underline{\underline{)}}_2 = (\underline{\underline{r}})_{2^2} = \underline{\underline{r}}^4 = \underline{\underline{r}}^{2 \times 2} = (\underline{\underline{r}}^2)_{2^2} = (\underline{\underline{r}} \oplus \underline{\underline{t}})_{2^2}^2$$

-- which connotes the 'self-reflexive function', or 'self-operation', of the *collective* dialectical operation -- of the *two-*category, *two-dimensional* categories-cumulum operation -- denoted by $(\underline{\underline{r}} \oplus \underline{\underline{t}})$.

Syntactically, per the axioms of the $n_{\mathbf{Q}}$ 'meta-numbers', we have the following values as the content of step 2:

$$\underline{\underline{)}}\underline{\underline{)}}_1 = (\underline{\underline{r}})_{2^2} = (\underline{\underline{r}} \oplus \underline{\underline{t}})_{2^2} = (\underline{\underline{r}} \oplus \underline{\underline{t}}) \otimes (\underline{\underline{r}} \oplus \underline{\underline{t}}) =$$

$$(\underline{\underline{r}} \oplus \underline{\underline{t}}) \oplus \cap (\underline{\underline{r}} \oplus \underline{\underline{t}}) = (\underline{\underline{t}} \otimes (\underline{\underline{r}} \oplus \underline{\underline{t}})) \text{ [using Miguel's shortcut] } =$$

$$\underline{\underline{q}}_r \oplus \underline{\underline{q}}_{rr} \oplus \underline{\underline{q}}_{tr} \oplus \underline{\underline{q}}_{tt} = \underline{\underline{r}} \oplus \underline{\underline{t}} \oplus \underline{\underline{q}}_{tr} \oplus \underline{\underline{q}}_{tt} \text{ [---] } \underline{\underline{q}}_1 \uplus \underline{\underline{q}}_2 \uplus \underline{\underline{q}}_{2+1} \uplus \underline{\underline{q}}_{2+2}$$

$$= \underline{\underline{q}}_1 \uplus \underline{\underline{q}}_2 \uplus \underline{\underline{q}}_3 \uplus \underline{\underline{q}}_4.$$

Our challenge, for this step, is to "solve for" the *two* new categories, $\underline{\underline{q}}_{tr}$ and $\underline{\underline{q}}_{tt}$, that appear for the first time in this step, that is, to determine the meanings, the connotations, the definitions, of each of these new 'categorograms', as 'connotative entailments' of the already solved / known / assigned meanings / connotations / definitions of the 'categorograms' $\underline{\underline{r}} \equiv \underline{\underline{q}}_r$ and $\underline{\underline{t}} \equiv \underline{\underline{q}}_{rr}$.

So, let us apply the general "'canon of interpretation'" of such dialectical categories to the specific case of this example. Per that "'canon'", the 'categorogram' $\underline{\underline{q}}_{tr}$ connotes a new 'uni-category'/hybrid «arithmos» -- the synthesis category -- that results from the «aufheben» of the $\underline{\underline{q}}_r$ category/«arithmos», by the $\underline{\underline{q}}_r$ category/«arithmos». This often means a category/«arithmos» each of whose «monads» is a hybrid of the «monads» of the $\underline{\underline{q}}_r$ category/«arithmos» with the «monads» of the $\underline{\underline{q}}_{rr}$ category/«arithmos». Also per that "'canon'", a 'categorogram' like $\underline{\underline{q}}_{tr}$ might also connote "'conversion", of «monads» of the $\underline{\underline{q}}_r$ category/«arithmos», into «monads» of the $\underline{\underline{q}}_{rr}$ category/«arithmos».

Also, per that "'canon'", the $\underline{\underline{q}}_{tr}$ symbol should point to a category that is already known to us, that is part of our synchronic, present conceptualized experience of the totality of The Sciences, and to a category that is ontologically different than, and that is richer in determinations than -- that is "more determinate" than -- that is more "complex" and more 'thought-concrete' than, all of the categories evoked in the preceding steps.

And, per that "'canon'", a symbol like $\underline{\underline{q}}_{tt}$ connotes the new category/«arithmos» that results from the self-«aufheben» of the $\underline{\underline{q}}_{tr}$ category/«arithmos», and, quite often, that results from the self-«aufheben» 'meta-monadization' of some of the «monads» of the $\underline{\underline{q}}_{tr}$ category/«arithmos».

That is, each «monad», or unit, of the $\underline{\underline{q}}_{tt}$ «arithmos»/category should be a 'meta-«monad», or 'meta-unit', of «monads» or units of the $\underline{\underline{q}}_{tr}$ «arithmos» / category.

So, the units of the $\underline{\underline{q}}_{tt}$ assemblage should be, in this specific case, 'meta-terms', 'terms of second degree', each one made up out of a heterogeneous multiplicity of 'terms of first degree'.

Also, the *category* q_{tt} should be *another category* -- *other* than *all* of the categories previously evoked, in the preceding steps; a '*contra-category*' of each of those categories, and a '*meta-category*'/'*meta-«arithmos»*' to the q_1 *category* /-«*arithmos*». Moreover, the q_{tt} symbol should point to a *category* that is already known to us, that is part of our *synchronic, present* conceptualized experience of the totality of *The Sciences*, and to a category that is ontologically different than, and that is richer in determinations than -- that is "more determinate" than -- that is more "complex" and more 'thought-concrete' than, all of the categories evoked in the preceding steps.

On the bases set forth above, we propose that our solution -- our 'semantification' -- of the syntactic term q_{tr} should be f , connoting the «*arithmos*»/*category* of Ockham's *propositions of the first intention*: $q_{tr} = q_f = f [\text{---}] q_3$.

We propose further that our solution/'semantification' of the syntactic term q_{tt} should be s , connoting the «*arithmos*»/*category* of Ockham's *propositions of the second intention*: $q_{tt} = q_s = s [\text{---}] q_4$.

In proposing these two 'semantifications', we are in resonance with Ockham in invoking here, *not* just a subdivision of the universe of terms of language, i.e., of *single words* of language, into two sub-universes, of "*categorematic*" words vs. of "*syncategorematic*" words, and with "*categorematic*" words corresponding to q_{tr} , and with "*syncategorematic*" words corresponding to q_{tt} , so that q_{tr} and q_{tt} would fail to differ, in terms of the *ontology* of their «*monads*», from q_1 .

We hold that the "'*conversion*'", i.e., the *apt description*, of the «*arithmos*»/*assemblage* of *physical "facts"*, by "*terms*", q_{tr} , as of the «*arithmos*»/*assemblage* of "*terms*", by "*terms*", q_{tt} , requires, in each case, something more than mere *single words*.

Human language is far more than the mere utterance of *single word*-sounds. Human language is, for starters, an «*arithmos*»/*assemblage* of *sentences* -- of *propositions*, that is, of well-formed '*meta-words*', each one made up out of a heterogeneous multiplicity of *words*.

We thus hold that q_{tr} must connote a multiplicity of "terms" -- both "*categorematic*" and "*syncategorematic*" alike -- combined, into *well-formed sentences*, i.e., into *propositions*, that describe our knowledge of the *physical world*, q_1 . So: $q_{tr} = q_f = f [\text{---}] q_3$. [cf. Boole's category of "Primary Propositions", *Laws of Thought*, Chapter IV, Proposition I.].

We thus hold also that q_{tt} must connote a multiplicity of "terms" -- both "*categorematic*" and "*syncategorematic*" alike -- combined, into *well-formed sentences*, i.e., into *propositions*, that describe our knowledge of the *secondary world* of our own creation, of that core component of 'The Human Phenome' which is *the world of human language itself*, q_4 . So: $q_{tt} = q_s = s [\text{---}] q_4$. [cf. Boole's "Secondary Propositions", *Laws of Thought*, Chapter IV, Proposition I.].

Thus, f connotes an «*arithmos*»/*assemblage* of "true" *proposition-units*, or of *proposition-«monads»*, that are *about* -- of *empirically-apt propositions* which *describe* -- *the physical world*.

And, s connotes an «*arithmos*»/*assemblage* of "true" *proposition-units*, or of *proposition-«monads»*, that are *about* -- of *cognitively-apt propositions* which *describe* -- *the world of human language*, as an expression of *the world of human thought*.

But the *assemblage* of "true" *propositions* about the "*real*" world, external to human thought, q_{tr} , is what "'*the sciences of the real*'" "contain"; is the "'content'" of those sciences; is what Ockham means by "'*the sciences of the real*'", by the category he names «*scientia realis*», which we will connote by l : $f \equiv l$.

And the assemblage of “true” *propositions* about the “*terms*” world, the world human language/thought, q_{tt} , is what “*the sciences of the rational*” “contain” for Ockham; is the “content” of those sciences; is what Ockham means by “*the sciences of the rational*”, by the category he names «*scientia rationalis*», which we will connote by n : $s \equiv n$.

Accepting these solutions, our step $s = 2$ *dialectical-model equation* thus becomes --

$$\begin{aligned} \text{)---(}_1 &= (\text{) } \underline{r} \text{(} \text{)}^2 = \underline{r}^4 = (\text{) } \underline{r} \oplus \underline{t} \text{(} \text{)}^2 = (\text{) } \underline{r} \oplus \underline{t} \text{(} \text{)} \otimes (\text{) } \underline{r} \oplus \underline{t} \text{(} \text{)} = \\ &(\text{) } \underline{r} \oplus \underline{t} \text{(} \text{)} \oplus \cap (\text{) } \underline{r} \oplus \underline{t} \text{(} \text{)} = (\text{) } \underline{t} \otimes (\text{) } \underline{r} \oplus \underline{t} \text{(} \text{)} \text{(} \text{)} \text{ [using Miguel's shortcut] } = \\ \underline{q}_r \oplus \underline{q}_{rr} \oplus \underline{q}_{tr} \oplus \underline{q}_{tt} &= \underline{r} \oplus \underline{t} \oplus \underline{\ell} \oplus \underline{n} \text{ [---] } \underline{q}_1 \text{ |} \oplus \text{ |} \underline{q}_2 \text{ |} \oplus \text{ |} \underline{q}_3 \text{ |} \oplus \text{ |} \underline{q}_4. \end{aligned}$$

-- and we have the meaning that, as of step 2 , $s = 2$, our model, $\text{)---(}_s = (\text{) } \underline{r} \text{(} \text{)}^2^s$, posits the *possible* existence of *four* philosophical categories -- (1) the «*arché*» category of *physical* “*r*ealities”, “plus”; (2) its ‘*first contra-category*’, of “*t*erms of language”, “plus”; (3) the ‘*first uni-category*’ of the preceding two categories, whose *units* or «*monads*» are, as *propositions*, *hybrids*, *combinations*, or *syntheses* of “categorematic” and “syncategorematic” *t*erms, and, in a sense, as “*synthetic*” [cf. Kant] *propositions*, *hybrids* of the *units* of *physical* “*r*eality” with the *units* of *t*erms, thus constituting the category of *propositions of the first intention*, constituting Ockham’s «*scientia realis*», “plus”; (4) the ‘*second contra-category*’, the category of *propositions of the second intention*, constituting Ockham’s «*scientia rationalis*», ‘*meta-monadic*’ with respect to the «*monads*» of the ‘*first contra-category*’, of “*t*erms of language”, given that these *propositions*, which are the *units* or «*monads*» of the «*scientia rationalis*», constitute, each one, a ‘*meta-term*’, or ‘*term of the second degree*’, each one made up out of a heterogeneous multiplicity of ‘*terms of the first degree*’, which are the *units* or «*monads*» of the *category*/*arithmos*/*assemblage* of *units* herein connoted by \underline{t} .

All *four* categories together, as an ‘*ideo-cumulum*’, comprise --

- (1) the «*physis*»-«*monads*», or ‘*fact-units*’, of the *physical* *r*ealities’ «*arithmos aisthetos*», or sensorially-perceived “*facts*”-*assemblage*, superposed with, and *named* -- in a *many-to-one* relationship -- *by*;
- (2) the ‘trans-Platonian’ «*arithmos eidetikos*» of *t*erms-*units*, or of *t*erms-«*monads*», *both* together superposed with;
- (3) the ‘trans-Platonian’ «*arithmos eidetikos*» of the *propositions-units*, or of *propositions*-«*monads*», of the Ockhamian «*scientia realis*», all *three* together superposed with;
- (4) the ‘trans-Platonian’ «*arithmos eidetikos*» of the *propositions-units*, or of *propositions*-«*monads*», of the Ockhamian «*scientia rationalis*».

¿But is there a *next* step, a step 3 , for this *classificatory*, *taxonomic*, *systematic-dialectic presentation* of *the Sciences*?

Our step 2 models where this *dialectic* completed for Ockham, and for his times; for what was *synchronic*/extant in his experience, and in the experience of his *contemporaries*.

Ockham saw, as far as we know, no other sub-categories of *Science* besides «*scientia realis*» & «*scientia rationalis*».

¿But is there another sub-category of *Science* that was *not synchronic*/extant for Ockham, but that *is synchronic* for us?

¿How could this now *four*-dimensional ‘possibility-space’ of categorial ‘ideo-ontology’ expand in a possible step 3 ?

Let’s see what arises if we iterate this model one more step, i.e., for $s = 3$.

Step 3. For step **s** = **3** of this *presentation-model*, since $2^3 = 2 \times 2 \times 2 = 8$, we have --

$$\underline{\text{)}-|-}_{3} = (\underline{\text{r}})^{2^3} = \underline{\text{r}}^8 = \underline{\text{r}}^{4 \times 2} = (\underline{\text{r}}^4)^2 = (\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}})^2$$

-- which connotes the '*self-reflexive function*', or '*self-operation*', of the *collective dialectical operation* -- of the *four*-category, *four*-dimensional *categories-cumululum operation* -- denoted by $(\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}})$.

Syntactically, per the axioms of the $\underline{\text{nQ}}$ 'meta-numbers', we have the following values as the content of step 3:

$$\underline{\text{)}-|-}_{3} = (\underline{\text{r}})^{2^3} = (\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}})^2 =$$

$$(\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}}) \otimes (\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}}) =$$

$$(\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}}) \oplus \sqcap (\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}}) =$$

$$(\underline{\text{n}} \otimes (\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}})) \text{ [using Miguel's shortcut]} =$$

$$\underline{\text{q}}_{\text{r}} \oplus \underline{\text{q}}_{\text{rr}} \oplus \underline{\text{q}}_{\text{tr}} \oplus \underline{\text{q}}_{\text{tt}} \oplus \underline{\text{q}}_{\text{nr}} \oplus \underline{\text{q}}_{\text{nt}} \oplus \underline{\text{q}}_{\text{nl}} \oplus \underline{\text{q}}_{\text{nn}} =$$

$$\underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}} \oplus \underline{\text{q}}_{\text{nr}} \oplus \underline{\text{q}}_{\text{nt}} \oplus \underline{\text{q}}_{\text{nl}} \oplus \underline{\text{q}}_{\text{nn}}$$

$$[\text{---}) \underline{\text{q}}_1 \text{ |}\pm\text{ } \underline{\text{q}}_2 \text{ |}\pm\text{ } \underline{\text{q}}_3 \text{ |}\pm\text{ } \underline{\text{q}}_4 \text{ |}\pm\text{ } \underline{\text{q}}_{4+1} \text{ |}\pm\text{ } \underline{\text{q}}_{4+2} \text{ |}\pm\text{ } \underline{\text{q}}_{4+3} \text{ |}\pm\text{ } \underline{\text{q}}_{4+4} =$$

$$\underline{\text{q}}_1 \text{ |}\pm\text{ } \underline{\text{q}}_2 \text{ |}\pm\text{ } \underline{\text{q}}_3 \text{ |}\pm\text{ } \underline{\text{q}}_4 \text{ |}\pm\text{ } \underline{\text{q}}_5 \text{ |}\pm\text{ } \underline{\text{q}}_6 \text{ |}\pm\text{ } \underline{\text{q}}_7 \text{ |}\pm\text{ } \underline{\text{q}}_8.$$

Our challenge, for this step, is to "solve for" the *four* new categories, $\underline{\text{q}}_{\text{nr}}$, $\underline{\text{q}}_{\text{nt}}$, $\underline{\text{q}}_{\text{nl}}$, and $\underline{\text{q}}_{\text{nn}}$, that is, to determine the meanings, the connotations, the definitions, of each of these new 'categorograms', as 'connotative entailments' of the already solved / known / assigned meanings or connotations of the 'categorograms' $\underline{\text{r}} \equiv \underline{\text{q}}_{\text{r}}$, $\underline{\text{t}} \equiv \underline{\text{q}}_{\text{tr}}$, $\underline{\text{l}} \equiv \underline{\text{q}}_{\text{tr}}$, and $\underline{\text{n}} \equiv \underline{\text{q}}_{\text{tt}}$.

Here, at this juncture, we wish to venture a conjecture as to the 'connotatively-entailed' meaning-solutions, or definition-solutions, of these *four* new categories, $\underline{\text{q}}_{\text{nr}}$, $\underline{\text{q}}_{\text{nt}}$, $\underline{\text{q}}_{\text{nl}}$, and $\underline{\text{q}}_{\text{nn}}$. In general, we see each of them as connoting the fruition of the *application* of the «*scientia rationalis*» -- i.e., of advanced *formal, mathematical logic / dialectic* -- to each of the already-defined domains, resulting in their "'*subsumption*'" by the «*scientia rationalis*».

We conjecture, more specifically, as follows --

- q_{nr} \equiv the «*scientia rationalis*» “‘conversion’” of the «*arithmos aisthetos*» of *physical facts*; the *rational-scientific* immanent *critique, explanation* [including *theory of* [past] *perceptual error*], and *reconstruction* of human perception of the «*realis*» realm, extending to the formation of a *universal, systematic-dialectical taxonomy* of this realm, related to the project of the «*Natur*» division of Hegel’s *Encyclopedia of the Philosophical Sciences*;
- q_{nt} \equiv the «*scientia rationalis*» “‘conversion’” of the «*arithmos eidetikos*» of *terms* of human language; the *rational-scientific* immanent *critique, explanation* [including *theory of* [past] *terminological error*], and *reconstruction* of human language, including the formation of a *rational universal language*, comprising a *dialectical phonetic language*, or ‘*dialectical phonogramy*’, as well as of a ‘*dialectical pictogramy*’, and of a ‘*dialectical ideogramy*’, related to Leibniz’s vision of a «*characteristica universalis*»; also related to the project of the «*Logik*» division of Hegel’s *Encyclopedia of the Philosophical Sciences*;
- q_{ne} \equiv the «*scientia rationalis*» “‘conversion’” of the «*arithmos*» of the *propositions* of the «*scientia realis*»; the *rational-scientific* immanent *critique, explanation* [including *theory of* [past] *scientific theories error / ideology*], and *reconstruction* of the «*scientia realis*», via the formation of ‘*theories*’ accounting for the *differences* in the history of *different theories/ideologies*, or *systems of propositions*, regarding *the same “real” facts / physical phenomena*, e.g., of *different theories/ideologies* of the same *facts* arising from different historical epochs/human-social formations, related to the project of the «*Natur*» division of Hegel’s *Encyclopedia of the Philosophical Sciences*, and to that of Marx’s *Grundrisse, A Contribution to the Critique of Political Economy*, and *Capital: A Critique of Political Economy*.
- q_{nn} \equiv the «*scientia rationalis*» “‘conversion’” of the «*arithmos*» of the *propositions* of the «*scientia rationalis*» themselves; the *rational-scientific immanent self-critique, self-explanation* [including *theory of* [past] *scientific theories error / ideology*], and *self-reconstruction* of the «*scientia rationalis*» by the «*scientia rationalis*» themselves, including the formation of ‘*meta-theories*’ accounting for the *differences* in the history of *different theories/ideologies*, or *systems of propositions*, regarding *the same mathematical* and other “*noetic*” *facts/phenomena*, e.g., of *different theories/ideologies* of “the same” “‘*noeta*’”, or objects of thought, as they arise in different historical epochs/human-social formations, extending to development of a comprehensive *mathematics of dialectics* itself; related to the project of Hegel’s *Science of Logic*, and especially to its second volume.

Note that Marx’s *A Contribution to the Critique of Political Economy*, and *Capital: A Critique of Political Economy*, provide a *specific* example, and, mostly implicitly, a *generic* paradigm, for a core component of q_{ne} as conjecturally defined above. Nothing like Marx’s works was extant in Ockham’s time, but those works *are* ‘*synchronic*’ for us, today. In those works, Marx addresses “...the evolution of the economic formation of society ... as a process of natural history” [Marx, Preface to the First German Edition of *Capital*], and, thus, addresses the human species and its ‘Phenome’ as a *natural object*, as a “*real*” *fact/object*, a part of the purview of «*scientia realis*». The subsidiary objects which Marx addresses in those two treatises -- values, *commodities, monies, capitals*, and their “law of [surplus-]value” -- are tied to *objects of physical “reality”* -- *packages of tea, bank notes, steam-powered looms*, etc. -- but they are *not* simply *physical objects*. They are also *noetic objects*, ‘meme[t]ic objects’, “‘mental objects’”, “‘ideal objects’”, ‘cultural objects’; ‘*psyche*-ic objects’, parts of the human-social “*intersubjectivity*”; parts of ‘The Human Phenome’ -- human, “social relations of production” [Marx].

Such ‘category-[idea[I]-]objects’, such ideas, such memes, residing within ‘The Human Phenome’, the very «*arithmos [eidetikos]*» of meme «*monads*», are “*material*” not only in the sense that they are *germane*, that they are *salient*, for *Science*, but also in that, being “*intersubjective*” -- held mentally, and behaviorally, *in common* among human actors -- they are also *objective*. Such is the paradigm that we call ‘*psychohistorical materialism*’, pioneered by Marx.

Note also that this conjectured solution models the research programme actually pursued by our co-founder, Karl Seldon.

The universe of discourse of our model of the Ockhamian *dialectic of Science*, as updated, for our time, above --

$$\underline{\text{H}}_3 = (\underline{\text{r}})^2 = \underline{\text{r}} \oplus \underline{\text{t}} \oplus \underline{\text{l}} \oplus \underline{\text{n}} \oplus \underline{\text{g}}_{\text{nr}} \oplus \underline{\text{g}}_{\text{nt}} \oplus \underline{\text{g}}_{\text{nl}} \oplus \underline{\text{g}}_{\text{nn}}$$

-- is that of 'the *elements* [*facts* and *words*], and the sub-divisions, of *knowledge* [*science*]'.

If we abbreviate this universe of discourse, restricting it to just "The Sciences", leaving its *elements* [*facts* and *words*] *implicit* therein, instead of making them *explicit*, as before, then perhaps we can encompass the most essential content of this *systematic dialectic*, or '*synchronic ideo-taxonomy*', by means of a *Triadic* Seldon Function of only three terms --

$$\underline{\text{H}}_1 = (\underline{\text{L}})^3 = \underline{\text{L}} \oplus \underline{\text{N}} \oplus \underline{\Psi}$$

-- instead of the earlier eight algebraic terms, but one whose third and final algebraic term, $\underline{\Psi}$, we must now define.

For the terms of this new model, we make the following assignments to, or interpretations of, the terms of the old --

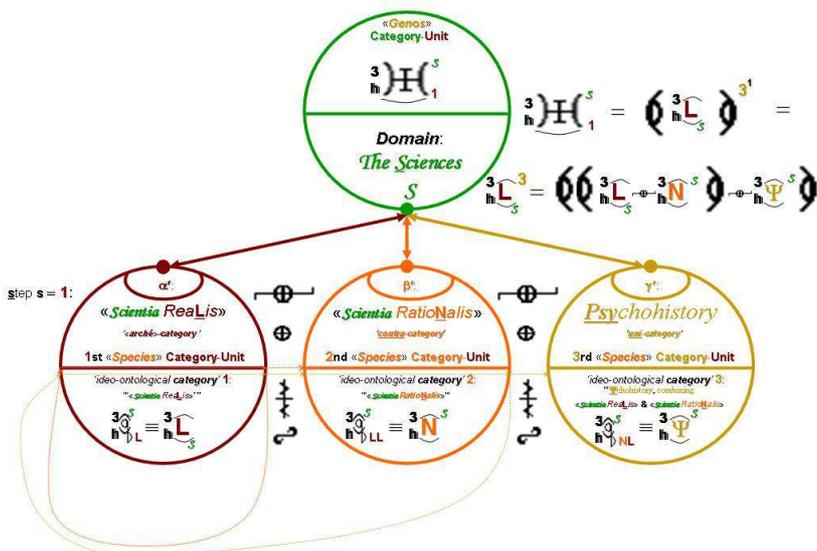
1. $\underline{\text{L}}$ (—) $\underline{\text{l}}$;
2. $\underline{\text{N}}$ (—) $\underline{\text{n}}$;
3. $\underline{\Psi}$ (—) $(\underline{\text{g}}_{\text{nr}} \oplus \underline{\text{g}}_{\text{nt}} \oplus \underline{\text{g}}_{\text{nl}} \oplus \underline{\text{g}}_{\text{nn}})$.

The '*cumulum*' -- the superposition -- of the conjectured definitions of the ideographical terms $\underline{\text{g}}_{\text{nr}}$, $\underline{\text{g}}_{\text{nt}}$, $\underline{\text{g}}_{\text{nl}}$, and $\underline{\text{g}}_{\text{nn}}$, taken together, amount to a definition of the *knowledge-field* which we name '[*socio-politico-economic-psychohistory*], or '*psychohistorical dialectics*', for short, and ' $\underline{\Psi}$ ' is our standard symbol for that *knowledge-field*. It is conjectured here as a "'*complex unity*'", or *dialectical synthesis*, of $\underline{\text{N}}$ & $\underline{\text{L}}$, of *logico-mathematical-cognitive-psycheological science* & *physical science*, including *human social science* as part of *the science of natural history* in Marx's sense: $\underline{\Psi} \equiv \underline{\text{g}}_{\text{NL}}$.

The fuller development of $\underline{\Psi}$ is the intended target and focus of the work of *Foundation Encyclopedia Dialectica*.

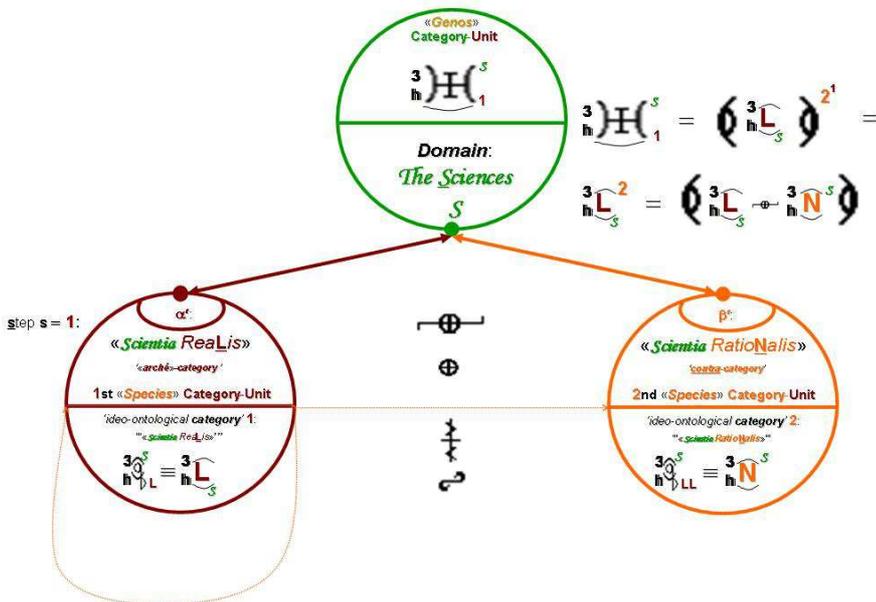
We can illustrate this proposed abbreviation, '*dialectical-pictographically*', as --

Triadic Seldon Function Systematic Dialectic of The Sciences [Extension of Ockham's *Dialectic of Science*]:
 '«arché»-category'; 'contra-category'; 'uni-category'

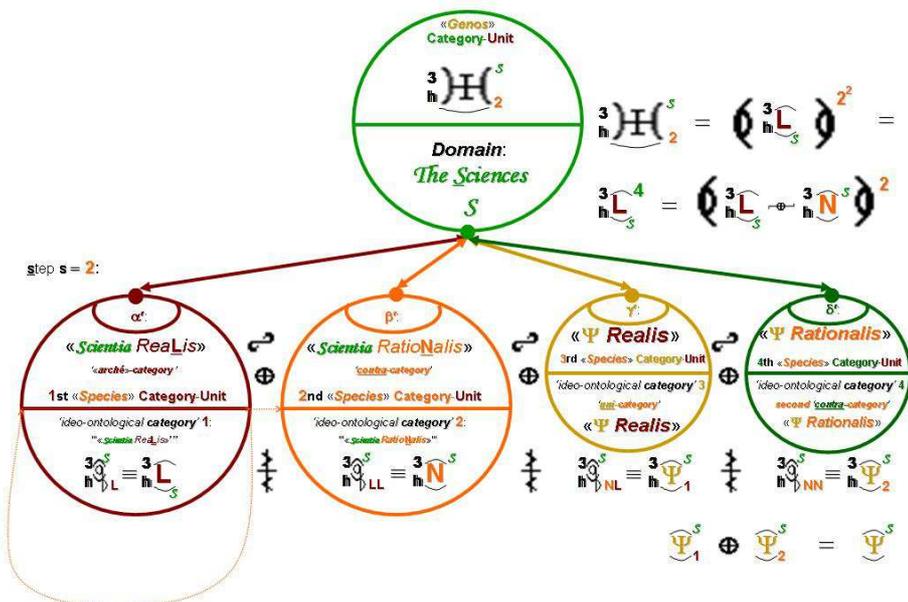


The connotations of this Ψ encompass the connotative content of the earlier model's algebraic terms g_{nr} , and g_{ni} , and also g_{nt} , since L , itself 'proxying' ℓ -- and as the operand of N , 'proxying' n , in the formation of Ψ -- implicitly connotes/'contains' the elements r and t . However, it can be well-argued that the 'self-reflexive moment' of the earlier model's $s = 3$ step -- namely, its final algebraic term, g_{nn} -- is missing from, this "abbreviated" model, which thus has a gaping 'homeomorphic defect' as a model of that earlier model. To get back this crucial component of the Seldonian concept of *psychohistory*, while still retaining some of the second model's succinctness, we can convert the *Triadic Seldon Function* model back into a *Dyadic Seldon Function* model, still using the same «arché». Then the new *dyadic* model's $s = 1$ step gives us Ockham's *dialectic*, & its $s = 2$ step gives the Seldonian 'extention', with $\Psi = \Psi_1 \oplus \Psi_2$, viz. --

Dyadic Seldon Function Systematic Dialectic of The Sciences, for presentation step $s = 1$:
 '«arché»-category'; 'contra-category'; 'uni-category'



Dyadic Seldon Function Systematic Dialectic of The Sciences, for presentation step $s = 2$:
 '«arché»-category'; 'contra-category'; 'uni-category'



V. Solution-Implicit Issue: Did Ockham Intentionally Construct a DIALECTIC of Science?

Ockham was certainly no stranger to concepts of “*the dialectic*”.

In the first place, medieval scholarship was permeated by notions of, and discourses upon, *dialectic*, to an extent which has yet to be fully explored and appreciated by modern scholarship.

In the introduction to her book *Dialectic and Its Place in the Development of Medieval Logic*, Eleonore Stump wrote:

“Since 1975 my work in medieval logic has concentrated on dialectic.”

“I have tried to trace scholastic treatments of dialectic to discussions of it in the work of Aristotle, the Greek commentators on Aristotle, and the Latin rhetorical tradition. But I have been especially interested Boethius [F.E.D.: 480-525 C.E.], whose discussions of dialectic were among the most important influences on scholastic treatments of the subject.”

“Accounts of dialectic based ultimately on Boethius’s views continued to play a fundamental role in philosophy through the fourteenth century.”

“The earliest scholastic logician whose work we know, Garlandus Compotista [F.E.D.: *Dialectica*, circa 1054 C.E.], devoted a great deal of attention to Boethian dialectic, and I have tried to follow the development of scholastic dialectic from Garlandus through various twelfth-century logicians (including Abelard) and thirteenth century terminists into the fourteenth century... ”

“The scholastic literature on dialectic is a vast treasure for scholarship, which even now is largely unmined”

[Eleonore Stump, *Dialectic and Its Place in the Development of Medieval Logic*, Cornell University Press [Ithaca: 1989], p. 1].

In the second place, Ockham himself was a major fourteenth century *innovative* contributor to medieval scholastic *dialectic*, principally via his circa 1323 treatise *Summa logicae*, or “*Summa[ry] of All Logic*”, including to the logic of “obligations”*. Some of his innovative contributions were reviewed by Eleonore Stump in the following terms --

“... there are also contexts (notably that of dialectical disputation, discussed in obligations treatises) in which types of inferences or rules for inferences that would otherwise be perfectly acceptable produce paradoxical results.”

“...for instance, Ockham discussed cases in which the otherwise acceptable rule ‘From the impossible anything follows’ does not hold.”

“In the burgeoning study of consequences and the growing interest in obligations the rather narrow formalization of logic brought about by the terminists was stretched and broadened. Ockham included a discussion of obligations within his treatment of inferences, and he also considered a broad variety of dialectical inferences.”

“In fact, he blurred the boundary between dialectic and demonstration, thereby elevating dialectic above the second-class status accorded it by the terminists.”

-- And --

“...contrary to the views of some contemporary scholars, there is nothing in Ockham’s criteria for valid inferences which corresponds to the notion of material implication in contemporary analytic philosophy.”

“... syllogisms, Topics, obligations, and insolubles are woven together in Ockham’s work into one general account of inferences which is at once richer and broader than that of the thirteenth-century terminists.”

“The terminists tend to force all nonsyllogistic arguments, including dialectical arguments, into syllogistic form. By contrast, Ockham’s account of inferences recognizes a variety of nonsyllogistic inferences, including dialectical and obligational inferences, without trying to mold them into syllogistic form; and it includes syllogisms as one species of consequences among others. The result, which clearly represents an advance, is a development toward a logic which is both formal and able to account for many sorts of inferences.” [Eleonore Stump, *ibid.*, pp. 3: 8-9].

*[We have an “obligation” here to define “obligation” as a term of logic. A respondent in a formal disputation is said to hold an *obligation* to avoid being forced into formally contradictory [“impossible”] assertions *due to respondent logical error*, given that the original assertion upheld by the respondent is a “possible” one.]

However, Ockham's work on dialectics was informed primarily by the dialectics of Aristotle, specifically that of Aristotle's Topics and Rhetoric, and thus only indirectly, not directly, by the dialectics of Plato.

We therefore cannot infer, with any certainty, that Ockham consciously intended his 'ideo-taxonomy' of *the Sciences* to be anything like a Platonian «*arithmos eidetikos*», especially given Ockham's "nominalist" views, rejecting Platonian and other hypostatizations or reifications of "universals".

More likely, Ockham's 'ideo-«*genos*»' of *the Sciences*, and its two 'ideo-«*species*»' of «*scientia realis*» and «*scientia rationalis*», were conceived within the prevailing Aristotelian «*genus*»/«*species*» 'ideo-taxonomical' tradition, which, though distantly related to Plato's «*arithmoi eidetikoi*» dialectic, represents a very attenuated form thereof, in the aftermath of Aristotle's critique and critical rejection of Plato's «*arithmoi eidetikoi*».

Links to definitions of additional ***Encyclopedia Dialectica*** special terms deployed in the discourse above --

«***arithmos aisthetos***»

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/ArithmosAisthetos/ArithmosAisthetos.htm>

«***arithmos eidetikos***»

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/ArithmosEidetikos/ArithmosEidetikos.htm>

categorial

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Categorial/Categorial.htm>

category

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Category/Category.htm>

‘***cumulum***’

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Cumulum/Cumulum.htm>

dialectical categorial progression presentations

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/CategorialProgression/CategorialProgression.htm>

homeomorphic defect

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/HomeomorphicDefect/HomeomorphicDefect.htm>

[***The***] ***Human Phenome***

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/PsychoHistory/PsychoHistory.htm>

immanent

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Immanent/Immanent.htm>

immanent critique

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/ImmanentCritique/ImmanentCritique.htm>

«***monad***»

<https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Monad/Monad.htm>

ontological category

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/CategoryOntological/CategoryOntological.htm>

ontology

<https://www.point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Ontology/Ontology.htm>

psychohistory

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/PsychoHistory/PsychoHistory.htm>

psychohistorical dialectics

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/PsychohistoricalDialectics/PsychohistoricalDialectics.htm>

qualo-fractal

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/QualoFractal/QualoFractal.htm>

qualo-Peanic

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/QualoPeanicity/QualoPeanicity.htm>

Seldon Functions

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/SeldonFunctions/SeldonFunctions.htm>

‘***self-meta-monad-ization***’ or ‘***self-meta-individual-ization***’ or ‘***self-meta-holon-ization***’

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Meta/Meta.htm>

<http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/MetaMonadization/MetaMonadization.htm>