The Dialectica Manifesto:

Dialectical Ideography

and

the Mission of F.*E*.*D*.

by

Foundation *Encyclopedia Dialectica*

The *Dialectic* of the *Immanent Critique* of *Set Theory*.

Work by a growing cadre mathematicians, beginning from the mid-**1800**s, sought to establish a Platonian view of mathematical ideas, today often called "Mathematical Platonism", or "Mathematical Realism".   This is the doctrine that mathematical objects are ***not*** human mental constructs. This doctrine holds that they are, instead, ***Real***, immutable, objective but ***transcendental*** entities, not accessible to sensuous perception, only to noetic experiencing. This doctrine asserts the existence of an eternal realm of '''mathematical «*eide*»''', but often without any inheritance of Platonian *dialectic*: a kind of partially, logically '*Aristotelianized Platonism*'.

Much of this work focused later on the development of "Set Theory", taken to be a theory of the ultimate, "Platonically Real" objects of mathematics.

This "Platonic" movement within mathematics was often tied to, or part of, a larger movement, sourced in an 'intendedly' ***non***-Hegelian, ***non***-dialectical logic, or even in a polemically and overtly ***anti***-dialectical logic.

This larger movement sought to '''mathematicize''' ***Aristotelian*** logic, and to produce an extended Aristotelian "mathematics of logic" that would also be the "logic of mathematics".

They then hoped to use that "mathematical logic", "symbolic logic", or '''ideographical logic''' to establish a secure, "axiomatic" and 'postulational' "Foundation" for ***all*** mathematics, *á la* Euclid's five-postulate deductive system for just the classical geometry ***part*** of mathematics.

Some within this movement even sought to "reduce" all of mathematics formal logic alone!

In either case, this would require the formulation, in the "artificial language" of that new "symbolic logic", of a few -- supposedly "self-evident", uncontrovertible -- premises, from which all of [the rest of] mathematics potentially could be, and then, slowly and painstakingly, would actually be, rigorously deduced.

Tendentially, as these efforts were pursued, the development of "Set Theory", of the new "Mathematical Logic", and of the 'aspirationally' secure logical "Foundation" for all of mathematics, became increasingly convergent and intertwined.

Among those who supported and contributed to at least certain aspects of this larger movement, we include Boole, Peirce, Cantor, Frege, Peano, Russell, and Gödel -- names which will arise again in the course of our outline, in this manifesto, of a dialectical, immanent critique of "Modern Mathematics" as a whole, a critique which parallels, in many ways, Marx's dialectical, immanent critique of "Modern [i.e., capital-epoch] Economics".

Crucially -- for their story, and for our story, and for our immanent critique -- their mathematical logic of the '[proto-]«*arithmoi*» *theory*', the [proto-]'*totality* *theory*', the "*ensembles* *theory*", the "*manifolds* *theory*", or of the "*set*-*theory*" approach to attaining an axiomatic foundation for all of mathematics created a model, and a kind of *metric*, for '''*sets within sets*'''.

The process of forming '''*sets inside of sets*''' is a process which we of **F**.**E**.**D**. describe as one of the '*meta-monadizing*', '*meta-«monads»-creating*', or '*logical meta-individuals-creating*', and thereby also as one of a *neo-«arithmos»*-making, *new* '*ideo-ontology*'-making, qualitatively self-transforming '*self-internalization*', '*self-re-entry*', '*self-inclusion*', '*self-incorporation*', '*self-containment*', or '*set-containment*', **of** *sets*.

It is a process in which *sets* themselves become their own *"opposites"* -- *elements* of *sets*.

This process of **the becoming**-*"elements"* of *"sets" themselves*; of **the becoming**-*"elements"* of *set* [**idea**[**l**]-]*objects, of entities which are* already *sets-of-elements*, is a process which we recognize to be one of '«*aufheben*» *self-subsumption*', as well as being a process which turns out to be **crucial** to the attempts of 'set-logicians' to "reduce" all of mathematics to 'set-logic'.

A *metric* for such *set* '*elementization*' is embedded in a theory called **the theory of logical types**.

A *set-representation* which *"contains"* only representations of *"logical individuals"*, e.g., of '*fundamental objects*', or '''*ur-objects*''', which are **not** themselves *sets*, might be assigned to '*logical type* **1**'.

Thus, for example, if **a** and **b** denote two such "thought-**concrete**", or *"determinations-rich"*, '*base*-[*idea*-]*objects*' [perhaps, at root, idea-representations of physical, sensuous objects], then the *set* denoted **{a, b}** -- the "collecting" or "gathering together" of the two [idea-]*objects* into a single ideal *unity* -- is then of **logical type 1**.

This *set*, "enclosing", or "containing", both **a** and **b**, thereby represents a more '*determinations-reduced*', '*characteristics-impoverished*', "more **abstract**" [*idea*-]*object*, because it is defined as denoting **only** those determinations, characteristics, qualities, or "predicates" which **a** and **b** **both** exhibit; which they "have in common".

A set of **logical type 2** would then be *a set that includes sets of '*base-objects*' among its* **elements**, such as the set denoted by:

$$\{ \mathbf{a}, \mathbf{b}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{a}, \mathbf{b}\} \}.$$

The '''logical type''' of a set, per the definition of '''logical type''' given above, can be determined directly by counting the number of '''opening braces''', '**{**', or of '''closing braces''', '**}**', to their deepest, or maximal, level within the 'braces-representation' of the set whose '''logical type''' metric is to be evaluated.

Notice that the contents of the set **{a, b}** are also [«*aufheben*»] contained/conserved within the contents of the set **{ a, b, {a}, {b}, {a, b} }**, but also that **{ a, b, {a}, {b}, {a, b} }** is a kind of **not**-**{a, b}** --

$$\{\mathbf{a}, \mathbf{b}\} \quad \neq \quad \{ \mathbf{a}, \mathbf{b}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{a}, \mathbf{b}\} \}.$$

Indeed, **{ a**, **b**, **{a}**, **{b}**, **{a**, **b} }** is *qualitatively unequal to* — **not** merely *quantitatively unequal to* — **{a**, **b}**:

$$\textbf{\{a, b\}} \quad \not\Vdash \quad \textbf{\{ a, b, \{a\}, \{b\}, \{a, b\} \}}$$

AND

$$\textbf{\{a, b\}} \quad \neq\!\!\!= \quad \textbf{\{ a, b, \{a\}, \{b\}, \{a, b\} \}}$$

AND

$$\textbf{\{a, b\}} \quad \not\Vdash \quad \textbf{\{ a, b, \{a\}, \{b\}, \{a, b\} \}}$$

THEREFORE

{a, **b}** is neither *quantitatively unequal* to, nor *quantitatively equal* to **{ a**, **b**, **{a}**, **{b}**, **{a**, **b} }**

ERGO

$$\textbf{\{a, b\}} \quad \text{is } \textit{qualitatively unequal} \text{ to } \quad \textbf{\{ a, b, \{a\}, \{b\}, \{a, b\} \}}$$

ERGO

$$\textbf{\{a, b\}} \quad \not\equiv \quad \textbf{\{ a, b, \{a\}, \{b\}, \{a, b\} \}}$$

-- wherein the new, '''**non-standard**''' relation-symbol, "'coined" by Karl Seldon, ' $\not\equiv$ ', enables

us to summarize, in a single statement, the 'negated trichotomy' of the conjunction of the three statements '**{a**, **b}** is *not greater than* **{ a**, **b**, **{a}**, **{b}**, **{a**, **b} }**', and '**{a**, **b}** *is not equal to* **{ a**, **b**, **{a}**, **{b}**, **{a**, **b} }**', and '**{a**, **b}** *is not less than* **{ a**, **b**, **{a}**, **{b}**, **{a**, **b} }**'.
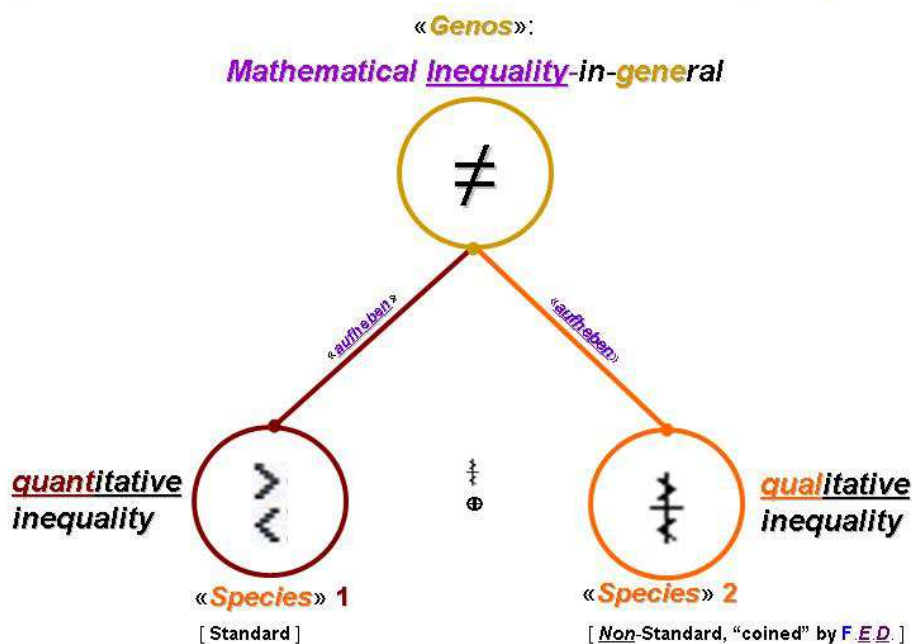

What we are saying, in other words, is that mathematics *immanently* needs to recognize, and distinguish, [at least] *two* qualitatively distinct «*species*» of the «*genos*» — denoted ''≠'' — of *inequality*.

One «*species*» is already recognized, and denoted herein, -- by the ideographical symbol ' $\gtrless$ '.

The other «*species*» is currently, in general, *un*recognized in conventional mathematics, and is denoted, herein by the "compound" ideographical symbol, and 'neogram' -- ' $\not\equiv$ '.

This *dialectical* «*diairesis*», or internal division, within the category of the mathematical relation of inequality, is illustrated below --

'''**Spec**iation''' of the «*Genos*» of the **Mathematical Inequality Relation**

«*Genos*»:

Mathematical Inequality-*in-general*

$\neq$

«*aufheben*»   «*aufheben*»

quantitative inequality

$>$
$<$

«*Species*» 1

[Standard]

qualitative inequality

«*Species*» 2

[*Non*-Standard, "coined" by F.*E*.*D*.]

'Trans-Platonian' «*Arithmos Eidetikos*» «*Aufheben*» Diagram:
The Two «*Species*» of the «*Genos*» of the **Mathematical Inequality Relation**

This relation, of '''qualitative inequality''', or of 'ontological inequality', is a key to the construction of a **non-reductionistic**, '''*holistic notation*''', in the later, higher '''dialectical arithmetics''' [which are evoked in a '''systematic-dialectical''' arithmetical model, **&** method of presentation, of those arithmetics], via the 'syntactics' of their modeling of the dynamics **&** '*meta-dynamics*' of '*meta-super*-*systems*', miming the dialectical '''[*self*-]evolution''' **&** '[*self*-]*meta-evolution*' of such '*meta-super*-*systems*', via 'quanto-qualitative', 'dialectical-mathematical' formulae.

Notice also that the 'successor-set', **{ a, b, {a}, {b}, {a, b} }**, differs, 'contentally', from the 'predecessor-set', **{a, b}**, in that it contains — together with the 'predecessor-set' itself, **{a, b}** — also [most of] the ["standard"] "sub-sets" of that 'predecessor-set'.

That is, 'the successor-set', **{ a, b, {a}, {b}, {a, b} }**, contains [most of] the elements of most of the ["standard"] "set of all sub-sets" — i.e., the elements of [most of] the so-called "power-set" — of the 'predecessor-set', **{a, b}**, '''plus''' [or "Union", denoted '∪'] that 'predecessor-set' itself.

The ["standard"] "sub-sets" of **{a, b}** include the *"improper"* subset of **{a, b}** — none other than *the whole* of set **{a, b}** itself — so that the 'successor-set', **{ a, b, {a}, {b}, {a, b} }**, results from, in part, a '*self*-*internalization*' of the previous whole/entire set, or '''totality''', **{a, b}**, which '''now''' becomes a '''mere''' [new] *part inside* the new, expanded, 'ideo-ontologically' richer whole / '''totality''', **{ a, b, {a}, {b}, {a, b} }**.

Thus, the 'successor-set', here, is the 'predecessor-set' itself, '''plus''' the elements of [most of] the "power-set" of that 'predecessor' set.

The various parts of the 'successor-set', **{ a**, **b**, **{a}**, **{b}**, **{a, b}** }, might, for example, be interpreted as follows: '**a**' names a concrete, complex, 'full-determinations' 'prior-to-sets' '''ur-object''', as does '**b**', for a qualitatively distinct / other such object; '**{a}**' names a predicate formulated to express, as a univocal, singular quality / '''in-tension''', the total '''nature''' /-content / '''predicate''' *unique* to '**a**'; '**{b}**', in turn, names a predicate formulated to express, as a singular quality / '''in-tension''', the total '''nature''' / content *unique* to '**{b}**', and; '**{a, b}**' names a predicate formulated to express, as a singular quality / '''in-tension''', just those qualit(y)(ies) shared in common by '**a**' and '**b**' *alone* among the totality of '''ur-objects''' that constitute the *base* of the universe[-of-discourse] being modeled.

The set-succession — or «*aufheben*» set-progression — partially depicted here is thus one which models what we term a '*predico-dynamasis*', or '*qualo-dynamasis*', progressively conceptualizing — or lifting out of '''chaotic''' and '''inchoate''' implicitude; progressively 'explicitizing' — more and more predicates, so as to articulate ever-more distinctly and ever more concretely, '''for-themselves''', the richness of the determinations of that universe's '''ur-objects''', '''in-themselves'''. This progression constitutes, indeed, a schematic model of the order in which human knowledge itself grows regarding a newly-opened domain of knowledge, or universe-of-discourse, from its inception.

Thus, in summary, the 'predecessor-set' / logical-type, above, is «*aufheben*»-*conserved*, and also, simultaneously, «*aufheben*»-*elevated* [in logical type, as well as being *expanded* in contents-*ontology*], and thus also «*aufheben*»-*negated*/annulled/canceled/*qual*itatively-transformed, by this '«*aufheben*» self-product', or 'Power-Set Evolute Self-Product', of sets.

If we denote by **T**, and also by **S₀**, the "universal set", the set of *All* '''logical individuals''', or, i.e., the '''**T**otality''' of '''ur-objects''' that are part(s) of a given universe-of-discourse, and if we denote by **s[T]** the '**s**uccessor universe-set' of the 'predecessor universe-set', **T**, **&** if **P[T]** denotes the "set of all subsets", or "**P**ower-set", of the set **T**, then the formula for the product-rule just named above can be stated as follows --

$$\underline{s}[\,\underline{T}\,] \;\equiv\; \underline{T} \times \underline{T} \;\equiv\; \underline{T}^2 \;\equiv\; \underline{T} + \underline{\Delta}[\,\underline{T}\,] \;\equiv\; \underline{T} \,\cup\, P[\,\underline{T}\,]$$

-- or --

$$\underline{s}[\,\underline{S}_0\,] \;\equiv\; \underline{S}_0 \times \underline{S}_0 \;\equiv\; \underline{S}_0{}^2 \;\equiv\; \underline{S}_0 \cup \underline{\Delta S}_0 \;\equiv\; \underline{S}_0 \,\cup\, P[\,\underline{S}_0\,] \;\equiv\; \underline{S}_1$$

-- or, more generally, for the variable **τ** successively taking on the values **0**, **1**, **2**, **3**, **4**, **...**, as --

$$\underline{s}[\,\underline{S}_\tau\,] \;\equiv\; \underline{S}_{\tau+1} \;\equiv\; \underline{S}_\tau \times \underline{S}_\tau \;\equiv\; \underline{S}_\tau{}^2 \;\equiv\; \underline{S}_\tau \cup \underline{\Delta S}_\tau \;\equiv\; \underline{S}_\tau \,\cup\, P[\,\underline{S}_\tau\,]$$

-- or --

$$\underline{s}^\tau[\,\underline{S}_0\,] \;=\; \underline{S}_\tau \;=\; \underline{S}_0{}^{2^\tau}.$$

The resulting «*aufheben*»-progression of sets — namely, the set-sequence-containing the set<u>s</u> denoted by **{ S<u>τ</u> }** as **τ** successively takes on the values **0**, **1**, **2**, **3**, **4**, **...** — i.e., for the "Natural" ordinality, or order of progression, of the "Whole" Number value, **τ**, provides, especially for '''*realistic*''', *finite*, ''actually-constructed''' successive universes-of-discourse, a *propositionally* ***non***-self-contradictory, non-paradoxical model of the most central, most crucial

[idea-]object in all of set theory as such, the '''*set of all sets*'''.

$$2^{\underline{U}} \cdots\cdots\triangleright \quad \{\ \bullet\ \bullet\ \bullet\ \} \qquad 2^{\{\cdots\}}$$
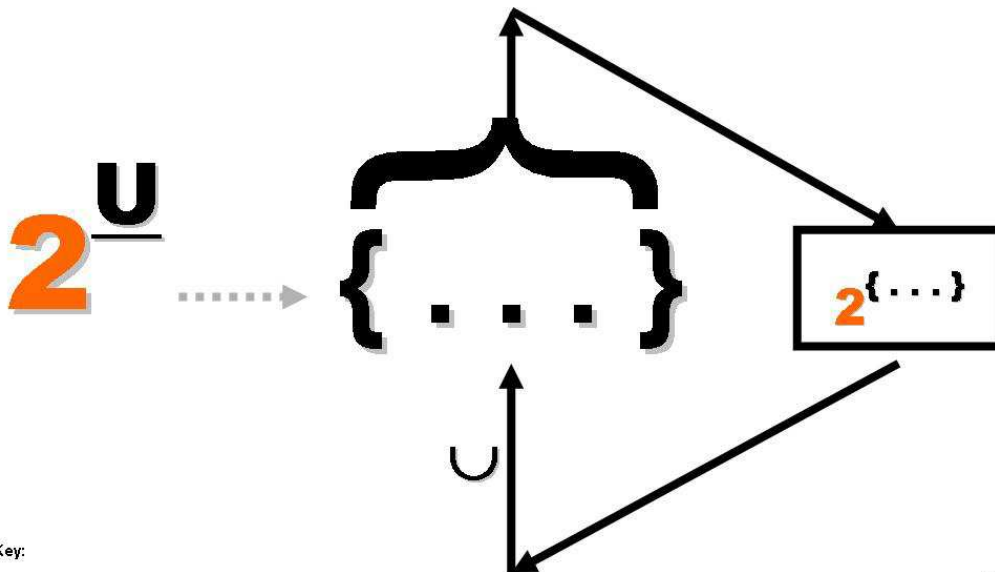
Key:

$\underline{U}$ = Finite [actually-constructed] **U**niversal Set, or set of all logical individuals, comprising the **U**niverse of Discourse in question; «**arché**»: $\underline{S}_0 = 2^{\underline{U}}$.

$2^{\{\cdots\}}$ = "Power-Set", or "Set of all Subsets", of the Set denoted by $\{\ \bullet\ \bullet\ \bullet\ \}$.

$$\underline{S}_{\tau+1} \equiv \underline{S}_\tau \cup 2^{\underline{S}_\tau}$$

«**Aufheben**» Diagram: The **Dialectical** 'Meta-Monadology' of the '''**Set of All Sets**'''

The «**arché**» set-**unit** here, the "**U**niversal Set", denoted by $\underline{U}$, must internalize its own "improper" subset, itself, plus all of its other -- "proper" -- subsets as well, to produce its try at the '''**Set of All Sets**''', whose result, the set-**unit** $\underline{S}_1$, must then, in turn, also internalize itself and its other subsets to make its next try at the '''**Set of All Sets**''' ...

The mental '''**eventity**''', '''**self-movement**''', or ‹‹**autokinesis**›› that is the **Set of All Sets** may be modeled by a '**Seldon Function**', i.e.,

$$s^\tau[\underline{S}_0] = [\underline{S}_0]^{2^\tau}, \text{ wherein } \underline{S}_k \times \underline{S}_k = \underline{S}_k{}^2 = \underline{S}_k \cup 2^{\underline{S}_k} = \underline{S}_{k+1}.$$

$\underline{S}_6 = [\underline{S}_0]^{2^6} = [\underline{S}_0]^{64} = \underline{S}_5 \times \underline{S}_5 \equiv \underline{S}_5 \cup P[\underline{S}_5] \longrightarrow$ $\boxed{\underline{S}_5} \overset{\ddagger}{\Rightarrow} \boxed{P[\underline{S}_5]}$

$\underline{S}_5 = [\underline{S}_0]^{2^5} = [\underline{S}_0]^{32} = \underline{S}_4 \times \underline{S}_4 \equiv \underline{S}_4 \cup P[\underline{S}_4] \longrightarrow$ $\boxed{\underline{S}_4} \overset{\ddagger}{\Rightarrow} \boxed{P[\underline{S}_4]}$

$\underline{S}_4 = [\underline{S}_0]^{2^4} = [\underline{S}_0]^{16} = \underline{S}_3 \times \underline{S}_3 \equiv \underline{S}_3 \cup P[\underline{S}_3] \longrightarrow$ $\boxed{\underline{S}_3} \overset{\ddagger}{\Rightarrow} \boxed{P[\underline{S}_3]}$

$\underline{S}_3 = [\underline{S}_0]^{2^3} = [\underline{S}_0]^{8} = \underline{S}_2 \times \underline{S}_2 \equiv \underline{S}_2 \cup P[\underline{S}_2] \longrightarrow$ $\boxed{\underline{S}_2} \overset{\ddagger}{\Rightarrow} \boxed{P[\underline{S}_2]}$

$\underline{S}_2 = [\underline{S}_0]^{2^2} = [\underline{S}_0]^{4} = \underline{S}_1 \times \underline{S}_1 \equiv \underline{S}_1 \cup P[\underline{S}_1] \longrightarrow$ $\boxed{\underline{S}_1} \overset{\ddagger}{\Rightarrow} \boxed{P[\underline{S}_1]}$

$\underline{S}_1 = [\underline{S}_0]^{2^1} = [\underline{S}_0]^{2} = \underline{S}_0 \times \underline{S}_0 \equiv \underline{S}_0 \cup P[\underline{S}_0] \longrightarrow$ $\boxed{\underline{S}_0} \overset{\ddagger}{\Rightarrow} \boxed{P[\underline{S}_0]}$

$\underline{S}_0 = [\underline{S}_0]^{2^0} = [\underline{S}_0]^{1} = \underline{S}_0 \longrightarrow$ $\boxed{\underline{S}_0}$

$P[\underline{S}_0]$ stands for the "**P**ower Set", or set of all **sub**sets, of the Set $\underline{S}_0$, often also expressed as $2^{\underline{S}_0}$.

«**arché**»: **set of all subsets** of $\underline{U}$, the "**U**niversal Set", or the set of all objects/"logical individuals", of a given **U**niverse of Discourse, denoted by: $2^{\underline{U}} \equiv \underline{S}_0 \equiv P[\underline{U}]$.

$\boxed{\underline{U}}$

A set of [**idea**-]objects / "elements" / "members" which **are**, in general, **sets**, or set-objects.

A set of [**idea**-]objects / "elements" / "members", or of "logical individuals" which are **not**, in general, **sets**.
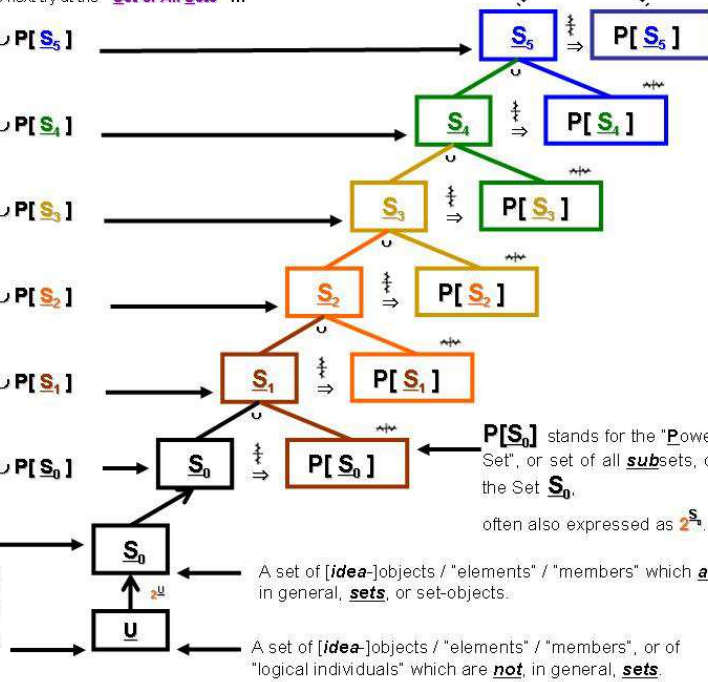
This '''*set of all sets*''' — since it is set-theory's own, native definition of the "set" itself, the set-theoretical, or "ex-tension-al", definition of the '''in-tension''' of the "set" concept itself — is the central idea-object of set-theory, though, ironically, and tellingly, it is **_suppressed_** in "Standard" Set Theory.

Hence, also, the '''*set of all sets*''' is the central locus of a **_dialectical_**, **_immanent_** *critique of* that set theory.

This '''*set of all sets*''' is a 'contra-Parmenidean' mental '*eventity*'.

It is a mental '''*self-movement*'''; an '*ideo-auto-kinesic*', '[*ideo-onto-*]*dynamical*', and '*ideo-onto-logic-ally*' *self-expanding* '''idea-object''', and one which, for appropriate universes-of-discourse, implicitly contains all of the content of what we call '*The Gödelian Dialectic*' [see next section].

¿But **_why_** is this '''*set of all sets*''' a 'self-changing' '''idea-object'''; an '''idea-object''' that itself induces change in itself; an '''idea-object''' that itself causes itself to expand, qualitatively, 'ideo-ontologically'; an idea-*object* that is also an 'idea-*subject*', or *agent of change*, with respect to itself; an 'idea-entity' that "won't stand still" in your mind, in any human mind, once that mind constructs it, and lends that mind's own 'subject-ivity' to that mental construct; an 'idea-entity' that forces itself to grow, and that is, thus, an 'idea-*event*ity', a mental process object "made of" '*ideo-*«*auto-kinesis*»'**?**

This [finitary] '''*set of all sets*''' is '''forced''', in an attempt to fulfill its own definition, the definition of its very self, i.e., in an attempt to "be[come]" what it "is" — viz., that it contains "**_All_**" sets — to **_force itself_** to continually expand of its contents, its '''elements''', its '''membership''' — to **_force itself_** into continual qualitative, ideo-ontological, 'predicatory' self-expansion, not by the '*internalization*' of anything '''*external*''' to it, because it already contains all of the '''ur-objects''' / "logical individuals" that **_found_** and **_base_** the entire universe-of-discourse in question, but, rather, on the contrary, via the continual '*self*[-*and-other-subsets*]-*internalization*', the '*internalization*' of what is **_already_** '''internal''' to it, of what it already '''contains''' **_implicitly_**; the '*internalization*' of *itself as a whole* — of its own "*im*proper subset*" — as well as of all of the "**_proper_** subsets" of itself.

This '''*set of all sets*''' is '''forced''' to do so, to continually re-'''*internalize itself*''' by its own nature / essence / 'essence-iality' / essentiality / logical necessity; by its own '''*self*'''; by its own name/description/*definition*, i.e., by the '*intra-duality*', or '*self-duality*', or '*indivi*[*sible*]-*duality*', of its *every* momentaneous '''state''' of existence in the human mind — because it always, in every "moment", "still" *excludes* those very sets which constitute *its own* "power set", *its own* subsets, among which is that set which is *its own* "*im*proper" subset, namely, none other than *itself*.

But this '''*set of ALL sets*''', *as that*, *as such*, is **_not_**, per its very name/definition, **_supposed to exclude any_** [finite, '''constructible'''] *sets at all*.

Yet, each time it '*internalizes*' all of its subsets, including itself, it thereby transforms itself into a new, qualitatively different, 'ideo-*ontologically different*', i.e., qualitatively expanded, 'ideo-ontologically' expanded, set, with yet a *NEW*, different set of subsets — a qualitatively different "power-set" — all of whose subsets are thus **_not yet included_** in itself, among its "elements".

Therefore, it must, *each time* it tries to [re-]form itself, *'internalize'* its own subsets, including itself, again.

But, in so doing, *each time*, it changes itself again, thus bringing a new, different set of [its] subsets -- a new, more rarefied set of 'extensional predicates' -- into [potential] existence.

And so, it must actualize that potential existence, by self-/power-set-*'internalizing'* yet again... .

Indeed, one obtains an augmented version of *the same* *'ideo-«auto-kinesis»'* result, if one simply defines the *"universal set"* itself as *"the set of ALL OBJECTS"* [of the universe-of-discourse in question], provided that one grants that the more '''rarified''', more abstract mental objects — that the *'idea-object'* that is each subset, i.e., each *"extensional predicate"*, denoted "extensionally", per set theory, by the set of all objects that share the quality denoted by that predicate — are included among the *"objects"* referenced by the sub-phrase *"ALL OBJECTS"*, **not** just the *"ur-objects"*.

One obtains, all over again, but this time in a deepened, more comprehensive form, a *mental process-object* characterized by self-expanding *'ideo-onto-dynamasis'*, in the form of an *'extensional-predicates-dynamasis'*, or *'predico-dynamasis'* --

The Finitary *Set of All* [*Idea-*]*Objects*: 'Self-Process' / *Ideo*-‹‹*Auto-Kinesis*›› Depiction

Key:
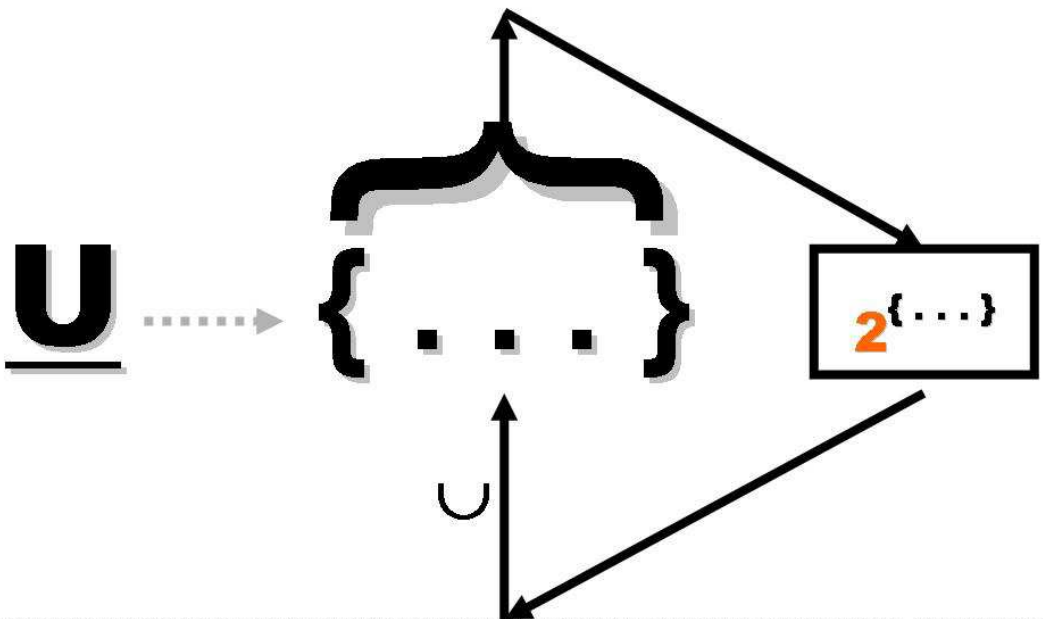
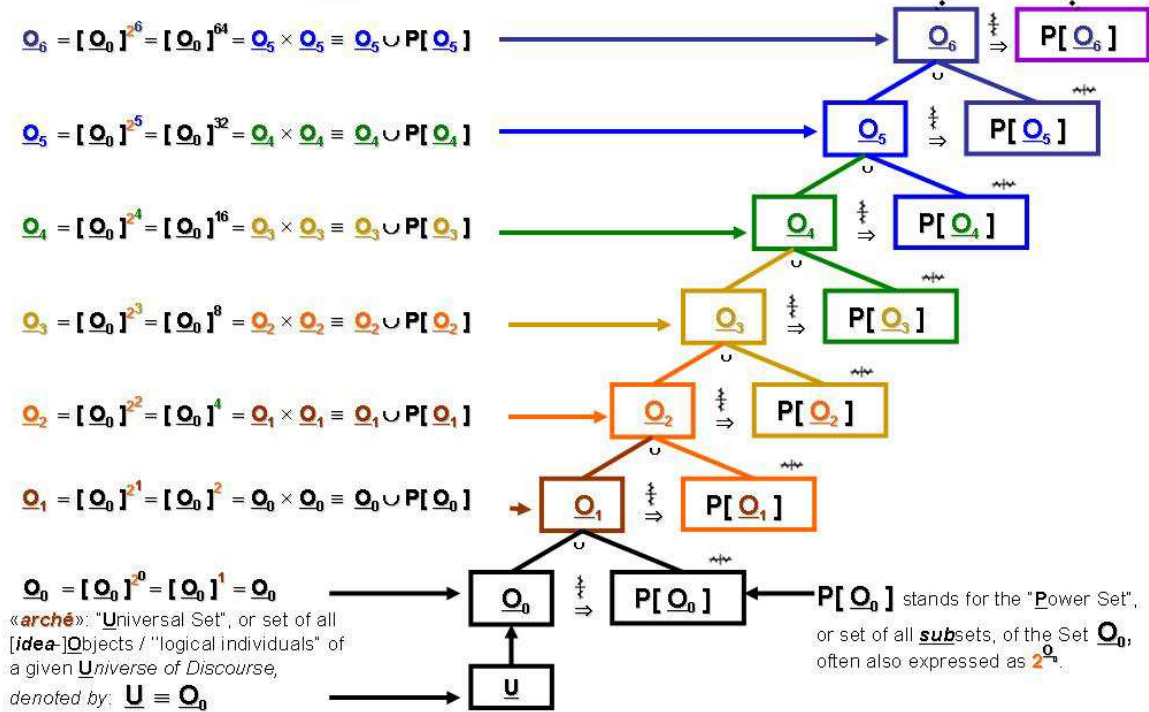$\underline{\mathbf{U}} \equiv \underline{\mathbf{O}}_0 \equiv$ Finite [actually-constructed] **U**niversal Set, or set of all logical individuals / set of all [*idea-*]**O**bjects, comprising the **U**niverse of Discourse in question.

$2^{\{...\}} \equiv$ "Power-Set", or "Set of all Subsets", of the Set denoted by $\{\ ...\ \}$.

$\underline{\mathbf{O}}_{\tau+1} \equiv \underline{\mathbf{O}}_{\tau} \cup 2^{\underline{\mathbf{O}}_{\tau}}$

«Aufheben» Diagram: The *Dialectical* 'Meta-Monadology' of the '''*Set of All* [*Idea-*]*Objects*'''

The «*arché*» set-*unit* here, the "Universal Set", denoted by $\underline{U}$, and also by $\underline{O}_0$, must internalize its own "improper" subset – itself -- plus all of its other -- "proper" -- subsets as well, to produce its next try at the '''*Set of All* [*Idea-*]*Objects*''', whose result, the set-*unit* $\underline{O}_1$, must then, in turn, also internalize itself and its other subsets to make its own, next, try at the '''*Set of All* [*Idea-*]*Objects*''', and so on...

The mental '''*eventity*''', '''*self-movement*''', or «*autokinesis*» that is the *Set of All* [*Idea-*]*Objects* may be modeled by a '*Seldon Function*',

i.e., $s^\tau[\underline{U}] = [\underline{U}]^{2^\tau}$, wherein $\underline{U} \times \underline{U} = \underline{U}^2 = \underline{U} \cup 2^{\underline{U}} = \underline{O}_1$.

$\underline{O}_6 = [\underline{O}_0]^{2^6} = [\underline{O}_0]^{64} = \underline{O}_5 \times \underline{O}_5 \equiv \underline{O}_5 \cup P[\underline{O}_5]$ → $\underline{O}_6 \overset{\ddagger}{\Rightarrow} P[\underline{O}_6]$

$\underline{O}_5 = [\underline{O}_0]^{2^5} = [\underline{O}_0]^{32} = \underline{O}_4 \times \underline{O}_4 \equiv \underline{O}_4 \cup P[\underline{O}_4]$ → $\underline{O}_5 \overset{\ddagger}{\Rightarrow} P[\underline{O}_5]$

$\underline{O}_4 = [\underline{O}_0]^{2^4} = [\underline{O}_0]^{16} = \underline{O}_3 \times \underline{O}_3 \equiv \underline{O}_3 \cup P[\underline{O}_3]$ → $\underline{O}_4 \overset{\ddagger}{\Rightarrow} P[\underline{O}_4]$

$\underline{O}_3 = [\underline{O}_0]^{2^3} = [\underline{O}_0]^{8} = \underline{O}_2 \times \underline{O}_2 \equiv \underline{O}_2 \cup P[\underline{O}_2]$ → $\underline{O}_3 \overset{\ddagger}{\Rightarrow} P[\underline{O}_3]$

$\underline{O}_2 = [\underline{O}_0]^{2^2} = [\underline{O}_0]^{4} = \underline{O}_1 \times \underline{O}_1 \equiv \underline{O}_1 \cup P[\underline{O}_1]$ → $\underline{O}_2 \overset{\ddagger}{\Rightarrow} P[\underline{O}_2]$

$\underline{O}_1 = [\underline{O}_0]^{2^1} = [\underline{O}_0]^{2} = \underline{O}_0 \times \underline{O}_0 \equiv \underline{O}_0 \cup P[\underline{O}_0]$ → $\underline{O}_1 \overset{\ddagger}{\Rightarrow} P[\underline{O}_1]$

$\underline{O}_0 = [\underline{O}_0]^{2^0} = [\underline{O}_0]^{1} = \underline{O}_0$ → $\underline{O}_0 \overset{\ddagger}{\Rightarrow} P[\underline{O}_0]$

«*arché*»: "Universal Set", or set of all [*idea-*]Objects / "logical individuals" of a given Universe of Discourse, denoted by: $\underline{U} \equiv \underline{O}_0$

$\underline{U}$

$P[\underline{O}_0]$ stands for the "Power Set", or set of all *sub*sets, of the Set $\underline{O}_0$; often also expressed as $2^{\underline{O}_0}$.

The '''*set of all sets*''' **is**, thus, a logical/conceptual/mental '*self-force*' that [en]forces the continual, mounting, *self*-«*aufheben*» '*self*-*internalization*' of itself together with its '*internalization*' of all of its [other] subsets, thus driving its qualitative self-expansion, in an open-ended, "potentially infinite" progress-of-knowledge 'meta-model'.

The '''*set of all sets*''' is, therefore —

(1) The very object which expresses and stands for the "essence" / "quality" that all sets have in common, per set theory's immanent way of expressing such qualities, such that, e.g., the number two is represented by the set of all sets which have exactly two members, and the color "green" is represented by the set of all objects that look green to human visual perception. However, contrary to the onto-*statical* proclivities of most "Standard" set-theorists, that quality turns out to be none other than an that of *an uninterrupted movement of self*-[*and-other-sub-sets-*]*inclusion*, of *self*-[*and-other-sub-sets-*]*subsumption*, of *self-involution*, of *self*-[*and-other-*]«*aufheben*» '*self*-[*and-other-sub-sets-*]*internalization*';

(2) The vehicle of an *immanent critique* of [Parmenidean] set theory itself, via a «*reductio ad absurdum*» refutation of Standard Set Theory's implicit 'Parmenidean Postulate' — the belief that sets, and their elements, and, indeed, that all mathematical-objects \ idea-objects, must be characterized by eternal «*stasis*», or changelessness;

(3) a set-theoretical model of the *'dialectic'* itself; of a generic *'Meta-Monadology'*; of what we will come to call, below, an 'auto-kinesic', 'ideo-onto-dynamical', *'Qualo*-Peanic', 'ideo-*meta-fractal'*-constructing, *'meta-finite'* 'self-progression'; an *'archeonic consecuum-cumulum'*, driven by a succession of *self*-«*aufheben*» *'self-internalizations'* which are also *'self-meta-*«*monad*»-*izations'*.

Sets of *logical type* **3** contain at most *sets* *of* *sets* *of* *base objects*, e.g. --

{ a, b, {a}, {b}, {a, b}, {{a}}, {{b}}, { {a, b} }, { {a}, {b} },... , { {a}, {a, b} }, { {b}, {a, b} } }.

Those elements of the latter set denoted by --

$$\textbf{\{ \{a\}, \{a, b\} \}} \text{ and } \textbf{\{ \{b\}, \{a, b\} \}}$$

-- are of a special kind of [sub-]sets, called *"ordered pairs"*, also written --

$$\textbf{⟨a, b⟩} \text{ and } \textbf{⟨b, a⟩}$$

-- respectively, because for them, *un*like for *sets* in general, *order of listing matters* --

$$\textbf{\{a, b\}} \;=\; \textbf{\{b, a\}}$$

-- but --

$$\textbf{\{ \{a\}, \{a, b\} \}} \;\equiv\; \textbf{⟨a, b⟩} \;\neq\; \textbf{⟨b, a⟩} \;\equiv\; \textbf{\{ \{b\}, \{a, b\} \}}$$

-- in fact, *in gene*ral [although *not* always so, e.g., when **a** and **b** stand for standard *numbers*] —

$$\textbf{⟨a, b⟩} \quad \text{'} \not\equiv \text{'} \quad \textbf{⟨b, a⟩}$$

-- wherein '≡' denotes "is equal to *by definition*".

Thus, if we take *"natural"* numbers to be our *'base* [idea-]objects', then sets or *"classes"* *"of"*, or "containing", such numbers would be of *logical type* **1**, *classes* *"of"* or "containing" *classes* [*of* such *numbers*] would be of *logical type* **2**, and *classes of classes of classes* [*of* such *numbers*] would be of *logical type* **3**, and so on.