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Subject: ***Preludes Series* – Prelude V.: *Solved Unsolvables***

Dear **www.dialectics.org Webmaster**,

Greetings to you from **Foundation *Encyclopedia Dialectica***!

Background. This letter contains **Prelude V**. of a series of Preludes to a forthcoming major manifesto by **Foundation *Encyclopedia Dialectica***. The series title is – ***Portents & 'Pre-Vestiges' of an Immanent Critique of the Ideology in Modern*, *'Mathematico-Science' as a Totality***. The series is based upon a sequence of commentaries already posted elsewhere on the World Wide Web. This **5**th **Prelude** is entitled: ***Solved Unsolvables***.

Prelude V.: *Solved Unsolvables*. There is a tendency in the literature of mathematics, physics, and other sciences today for writers to state that general ***nonlinear*** differential equations *"**cannot**"* be solved [*"**ever**"*].

This is an overstatement, an over-extrapolation from the more factual statement that, in general, most ***nonlinear*** differential equations ***have not*** [***yet***] been solved, though there are already a few -- very telling -- exceptions even to that already qualified statement.

To build your confidence in the progressive solvability of the once-unsolvable in human cognitive history, the story below takes you through a number of *"**un**solvable"* equations, ***and their solutions*** -- solutions that will, at least in the earlier cases, feel trivial to you, but that, as dramatized in the stories below, once gave even the most brilliant of our ancient ancestors a very hard time indeed. These equations truly were *"**un**solvable"* within the narrower perspectives that even the most brilliant of our ancestors once maintained.

1. **The Paradox of Gainless Addition**. The equation --

$[\,2 + x = 2\,]$ or $[\,x = 2 - 2\,]$

-- states a paradox: how can the addition of a[n unknown] number, here denoted by **x**, produce a result, a sum, that is ***not bigger*** than that 'known' number, here **2**, to which that "unknown" number, **x**, is added?

Given the "**N**atural Numbers", or **N**, «***genos***» of number, addition always means increase. In *that* context of human mental experience, addition *never* means *no* increase. The above-written equation is ***truly not solvable within*** the system of arithmetic called that of the cardinal, or sometimes, that of the "**N**atural", numbers --

$\mathbf{N} \equiv \{1, 2, 3, \ldots\}$.

However, the above-written equation ***is solvable***, by the 'non-diophantine number" **0**, within the 'ideo-ontologically' expanded system / space / set of the "**W**hole numbers" --

$\mathbf{W} \equiv \{0, 1, 2, 3, \ldots\}$.

Adjunction of the zero concept may seem trivial to us, yet it entailed a great and protracted conceptual travail for our ancient Mediterranean ancestors, and, with respect to issues surrounding division by zero, and the related issues of [especially] ***nonlinear*** differential equation ***singularity***, remains fraught with unresolved problems, "even" among we moderns today!

2. **The Paradox of Subtractive Addition**. The equation $[\ 2 + x\ =\ 1\ ]$ states a paradox: how can the addition of a[n unknown] number, **x**, produce a result, a sum, that is ***less than*** that 'known' number, here **2**, to which that "unknown" number, **x**, is added?

Within the **W** «***genos***» of number, addition always means a change that increases, or, at minimum, that results in no change at all, ***but it never means a decrease***.

The latter equation thus finds no number among the "**W**holes" to solve/satisfy it. It ***is truly unsolvable*** within the **W**hole Numbers. However, it ***is solvable*** within the 'ideo-ontologically' expanded number-"space" of the "integers", or "'integral'" numbers, the expanded numbers-set --

Z $\equiv$ **{..., –3, –2, –1, ±0, +1, +2, +3, ...}**.

The number-space, or number-set, standardly denoted by **Z**, is a qualitatively, that is, 'ideo-***ontologically***' ***expanded***, new-***kinds***-of-numbers-***expanded***, ***meaning***-of-number-***expanded***, or '***meme***[-ing]'-of-"number"-***expanded***, ***semantically-expanded*** universe-of-discourse of "Number", vis-à-vis the preceding «***genos***» of "Number", the **W** universe-of-discourse.

The equation $[\ 2 + x\ =\ 1\ ]$ is solved / "satisfied" by the 'non-diophantine number' **–1**.

3. **The Paradox of 'Decreasive' Multiplication**. Next, the equation --

$[\ 2 \times x\ =\ 1\ ]$, or, simply, $[\ 2x\ =\ 1\ ]$

-- also states a "'paradox'": how can the multiplication of any number, namely that of the "multiplicand", denoted here by the algebraic "variable" or "unknown"-symbol, **x**, by another, known, number, the "multiplier", produce a product which is ***less than*** that "multiplier", here **2**?

Multiplication, within the **Z** «***genos***» of number, always produces a 'product' which is either **(a.)** increased *in absolute value* relative to the "multiplicand" "factor", **(b.)** leaves the multiplicand unchanged, or **(c.)** turns it into zero. But **Z** multiplication can never turn a **2** into a **1**. Such an equation is ***truly not solvable within*** the system of arithmetic of the "integers", **Z**.

This equation is solvable, however, via 'ideo-***ontological expansion***' to encompass the qualitatively different system of arithmetic of the "**Q**uotient numbers", "ratio-numbers", "*ratio*-nal" numbers, or "fractions", denoted by **Q**, i.e., by an expansion that encompasses yet a ***new kind*** of number, the 'split a-tom' [the 'cut uncuttable'], the 'monad-fragment', or "fractional value", e.g., the number **+1/2**:

Q $\equiv$ **{....–2/1...–3/2...–1/1...–1/2...±0/1...+1/2...+1/1...+3/2...+2/1....}**.

4. **The Paradox of the *'Odd Ratio'* that Must Also be an *'Even Ratio'***. The [algebraically] ***nonlinear*** equation $[\ x \times x\ =\ 2\ ]$, or, simply, $[\ x^2\ =\ 2\ ]$, states a "'paradox'" too: it requires **x** to be of a kind of number which is, in some sense, '*both* [or *neither*] odd *and* [*nor*] even at the same time' [per the proof-strategy of the classic «***reductio ad absurdum***» demonstration of the "***ir***-ratio-nality" of the square root of **2**].

This equation is ***truly <u>not</u> solvable*** *'''**<u>ratio</u>**-nally'''*, i.e., is ***<u>not</u> solvable*** by any *'''**<u>ratio</u>**-nal'''* ***fraction***. It is solvable via 'ideo-***ontological***' ***expansion*** to the so-called "**<u>R</u>**eal" numbers, this time by two distinct numbers, given the algebraically ***<u>nonlinear</u>***, "**2**nd degree" character of this *"**<u>un</u>solvable**"* ***equation***, rather than by just one number, as were the preceding, [algebraically] linear, or **1**st degree, *"**<u>un</u>solvable**"* ***equations*** / *'''**paradoxes**'''*. This case may ***<u>not</u>*** seem trivial to you, unless you've already studied what we might term '''advanced arithmetic'''.

The two solutions are the "irrational" "**<u>R</u>**eal" values $-\sqrt{2}$ and $+\sqrt{2}$:

$$\mathbf{R} \equiv \{.....-\pi....-3....-e....-\sqrt{2}....-1....\pm 0....+1....+\sqrt{2}....+e....+3....+\pi.....\}.$$

5. **<u>The Paradox of the *Additive Inverse* = *Multiplicative Inverse* "Identity"</u>**. Finally, for the purposes of this tapestry of clues, the algebraically ***<u>nonlinear</u>*** equation $[\, x \times x + 1 = 0 \,]$, or, simply, $[\, x^2 + 1 = 0 \,]$ states a '''paradox''' as well: it implies that --

$$-x = +1/x$$

-- requiring a kind of number, whose *additive inverse*, $-x$, equals its *multiplicative inverse*, $+1/x$, or x^{-1}. This case also may not seem trivial to you, unless you've already studied what we call "advanced arithmetic". Among the so-called "**<u>R</u>**eal" numbers --

$-\pi \neq +1/\pi$,
$-3 \neq +1/3$,
$-2 \neq +1/2$, …, etc., etc. …

The equation $[\, x^2 + 1 = 0 \,]$ is ***truly <u>not</u> solvable***, ***<u>not</u>*** "***satisfiable***", within any of the foregoing «***gene***» of number, or of arithmetics, up through and including that of the so-called "**<u>R</u>**eal" numbers.

The equation $[\, x^2 + 1 = 0 \,]$ ***<u>is</u> solvable***, via an expansion of our number-***<u>kinds</u>*** 'idea-***ontology***' to that of the so-called "**<u>C</u>**omplex" numbers, denoted --

$$\mathbf{C} \equiv \{\, \mathbf{R} + \mathbf{R}\cdot\sqrt{-1} \,\}.$$

It ***<u>is</u>*** **<u>solvable</u>**, again -- and for the same reason as for the ***algebraically <u>nonlinear</u> equation*** $[\, x^2 = 2 \,]$ -- by **2** numbers, rather than just by **1** number, as would an algebraically ***<u>linear</u>***, or **1**st degree, equation. These two solutions -- these two numbers -- are standardly known as the "pure **<u>i</u>**maginary" numbers, "**<u>i</u>**maginary" numbers with "***<u>no</u>***" [with **0**] "**<u>R</u>**eal part". Using $r = +1$ to denote [the] "**<u>R</u>**eal unit[y]", they are --

$$x = +\sqrt{-1} = 0\cdot r + 1\cdot(+i) = +i,$$

and

$$x = -\sqrt{-1} = 0\cdot r - 1\cdot(+i) = -i.$$

The epithets "**<u>R</u>**eal" and "**<u>I</u>**maginary", in this context, can be misleading.

If "**<u>I</u>**maginary" means "conceptual", then the "**<u>R</u>**eal" numbers are no less "**I**maginary" than the "**<u>I</u>**maginary" numbers. If "**<u>R</u>**eal" means "experiential" then the "**<u>I</u>**maginary numbers" are "**<u>R</u>**eal" for modern experience as well, in that they encode the ubiquitous and permeating presence of the oscillations of electrical currents and of electromagnetic radiation -- of light, visible and invisible -- and also in terms of *"mathematical* experience", as a «***species***» of *"mental* experience*"*. The *"**Fundamental Theorem of Algebra**"*, which asserts that every algebraic equation can be solved, with as many solutions as the highest degree/power in which the unknown appears in the equation, is ***<u>not</u>*** true within the "**<u>R</u>**eal numbers", **R**. It ***<u>is</u>*** true within the "**<u>C</u>**omplex numbers", **C**.

Note how each successor «***genos***», or ***universe***[-of-discourse], of the number concept, cumulatively contains all of its predecessor universes of number, or is a "'***conservative extention***'" of all of its predecessor "'***universes***'", and, especially, of its immediate predecessor number system.

Such a cumulative, progressive ***'consecuum'*** of «***gene***» evinces part of the essence of what we mean by a ***'"dialectic"'***; by a ***'"dialectical"'***, or «***AUFHEBEN***», process, and by a ***'meta-dynamical***, ***meta-system-ic***, ***meta-evolutionary self-progression of systems'***, 'self-launching' from an originating, or «***arché***», system, here, the arithmetical system of the **N**, or "**N**atural", Numbers, and driven by ***the Gödelian movement from greater to lesser incompleteness*** -- i.e., by what we have termed ***'The Gödelian Dialectic'***.

The individual systems, in this cumulative progression of systems, are systems of arithmetic, mathematical systems, ***'idea***-systems', which exhibit this ***incompleteness-driven***, ***"unsolvability"-driven***, expansion of ***'idea*** *ontology'* -- of ***kinds of numbers*** *ontology*.

Other examples, as explored in the forthcoming Manifesto, will involve individual systems which are ***historical formations of human society***, or which are ***pre-human natural systems***, etc.

The amazing discovery here is that the same generic principles of ***cumulative***, «***aufheben***» [***self***-] ***progression*** -- i.e., of ***dialectic*** -- apply in all of these, seemingly disparate, cases.

But if that is true, ***how could and why should it be true?***

Dialogically yours,

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