F.*E*.*D*. Preface to *E*.*D*. *Brief #7*, on the $_{Z}Q$, by Guest Author "J2Y"

by *Hermes de Nemores*, General Secretary to the F.*E*.*D*. General Council

Commentary on *E*.*D*. *Brief #7*. Our new guest author, known pseudonymously as "Joy-to-YoU", and whom I shall reference herein, using the nickname with which he often references himself in our correspondence -- "J2Y" -- has provided to you, our readers, *a new and highly-accessible* «***entrée***» into the ***third*** ***stage*** within Q, the 'meta-system' of the F.*E*.*D*. '***First Dialectical Arithmetics***': namely, into the $_{Z}Q$ axioms-system of ***dialectical arithmetic***, with its *core* set, or space, of ***dialectical***, '*Integer-based, or* **Z**-*numbers-based, purely-**qual**itative* ***meta-numbers***' --

$$_{Z}Q \equiv \{\dots, q_{-3}, q_{-2}, q_{-1}, q_{\pm 0}, q_{+1}, q_{+2}, q_{+3}, \dots\}.$$

This new Brief, *E*.*D*. ***Brief #7***, caps a *trilogy* of Briefs prepared for you by J2Y, since late June **2012**, on the $_{N}Q$, the $_{W}Q$, and the $_{Z}Q$ ***dialectical arithmetics***, **&** their *exotic* arithmetical/algebraic *'ideo-ontology'* and *'ideo-phenomenology'*.

In each Brief of this trilogy, J2Y has alluded to the rising degree of "definiteness" -- of '''determinate-ness''', or of 'features-richness' -- expected to grow with every transition from term to '***Qualo***-Peanic' successor term in a ***dialectical categorial progression***, including in a ***dialectical*** [axioms-]***systems progression***, such as the one that J2Y has presented for you in his last three Briefs. We think the contents of these Briefs themselves provide specific "*self* evidence" of -- i.e., in themselves provide instantiation of, **&** data supporting -- this expectation regarding ***dialectical progression*** in general.

J2Y's ***Brief #5***, on the $_{N}Q$ system of ***dialectical arithmetic***, required **7** pages of text to achieve a satisfactory degree of specificity regarding that ***first*** system. His ***Brief #6***, on the $_{W}Q$ system, required **8** pages of content to satisfactorily cover the new features -- the $\Delta_{N}Q = {}_{A}Q$ incremental new *'ideo-ontology'* -- of that ***second*** system. His ***Brief #7***, on the $_{Z}Q$ system, took **19** pages of text to adequately address the new *'ideo-phenomena'* -- the $\Delta[{}_{N}Q \oplus {}_{A}Q]$ incremental new *'ideo-ontology'* -- of that ***third*** system. The escalation from **7** units to **8** units to **19** units -- using page units of expository text as a crude proxy for the 'features-richness' being exposited thereby and therein -- exhibits the kind of acceleration of "definiteness" to which J2Y often alluded therein.

What J2Y has accomplished for you, in *E*.*D*. ***Brief #7***, is to develop a *single* new *'''idea-object'''*, denoted C_{Z}, with which he shows how to *co-generate*, in a coordinated way, key new features of the $_{Z}Q$ axioms-system, which are not ["yet"] extant in the $_{N}Q$ axioms-system, or even in the $_{W}Q$ axioms-system. He does so by way of subsuming, into a "pure-**q**ualifiers" arithmetic, the *"purely-**quant**itative"* arithmetic of the Standard Integers, the new kind of [*"signed"*] numbers contained in the set, or space --

$$Z \equiv \{\dots -3, -2, -1, \pm 0, +1, +2, +3, \dots\}$$

-- *vis-à-vis* the **W** and the **N** number-spaces, showing how to ***unify*** some of the *amazingly* novel characteristics of the $_{Z}Q$ axioms-system of "purely-**q**ualitative", ***dialectical arithmetic***.

These novel features of $_{z}Q$, *vis-à-vis* $_{w}Q$, and $_{N}Q$, as well as *vis-à-vis* other, "standard", arithmetics, include --

1. Continuation of the "identity" of the ***additive*** *identity element* with the ***multiplicative*** *identity element*, which first emerged, as q_0, in $_{w}Q$, now in the form of $q_{\pm 0}$, in $_{z}Q$: $q_z + q_{\pm 0} = q_z = q_z \times q_{\pm 0} = q_z + q_{\pm 0} + q_{z \pm 0}$ $= q_z + q_z = q_z$ [using the **F**.*E*.*D*. '***meta-genealogical evolute product***' rule for $_{z}Q$ multiplication]; ...

2. Now with the added twist, in $_{z}Q$, for the first time, that ***additive*** *inverses* and ***multiplicative*** *inverses* are equal as well:

$$q_{+z} + q_{-z} = q_{\pm 0} = q_{+z} \times q_{-z} = q_{+z} + q_{-z} + q_{(+z)+(-z)} = q_{\pm 0} + q_{\pm 0} = q_{\pm 0};$$

3. Equivalent expressions of $_{z}Q$, generated by "revolving" signs around the q symbol as center, e.g., counter-clockwise:

$$-q_{+z}^{\ +1} = +q_{-z}^{\ +1} = +q_{+z}^{\ -1};$$

4. The emergence, in $_{z}Q$, for the first time, of what might have been expected to "wait" until $_{Q}Q$, namely, of '**q**ualifier fractions' with both '**q**ualifier numerators' and of '**q**ualifier denominators', and thus also of '**q**ualifier ratios', and of the '**q**ualifier division' operation, as a partial inverse operation of the $_{z}Q$ '**q**ualifier multiplication' operation, viz. --

- for all z in $\mathbb{Z}$: $q_{+z} = q_{+z}/q_{\pm 0} = q_{\pm 0}/q_{-z}$; $q_{-z} = q_{-z}/q_{\pm 0} = q_{\pm 0}/q_{+z}$, including $-q_{\pm 0} = +q_{\pm 0} = \pm q_{\pm 0}/\pm q_{\pm 0}$;
- for all z in $\mathbb{Z}$: $q_z/q_z = [q_z]^{+1} \times [q_z]^{-1} = [q_z]^{-1} \times [q_z]^{+1} = [q_z]^{\pm 0} = q_{\pm 0}$, including $[q_{\pm 0}]^{\pm 0} = q_{\pm 0}$;
- for all z in $\mathbb{Z}$: $[q_{+z}/q_{\pm 0}] \times [q_{\pm 0}/q_{+z}] = [q_{+z}/q_{+z}] = [q_z]^{+1-1} = [q_z]^{-1+1} = [q_z]^{\pm 0} = q_{\pm 0}$;
- for all j, k in $\mathbb{Z}$: $[q_k/q_j]^{-1} = [q_j/q_k]^{+1} = q_{+j} + q_{-k} + q_{+j-k}$; $[q_j/q_k]^{-1} = [q_k/q_j]^{+1} = q_{+k} + q_{-j} + q_{+k-j}$;
- for all z in $\mathbb{Z}$: $-1 \times q_{+z} = q_{-z}$; $-1 \times q_{-z} = q_{+z}$; $+1 \times q_{+z} = q_{+z}$, & $+1 \times q_{-z} = q_{-z}$;
- for all z in $\mathbb{Z}$: $\pm 0 \times q_z = q_{\pm 0}$, so $0 q_z = q_z^{\ 0} = q_0 \equiv \blacksquare$, called 'full zero', as distinct from 'empty zero', **0**.

For the **F**.*E*.*D*. research collective, this ***dialectical arithmetic***, $_{z}Q$, the ***third*** step in the '*meta-systematic meta-evolution*' ***within*** the **F**.*E*.*D*. '***First Dialectical Arithmetics***', Q, has always -- ever since it first emerged in our research -- held, for us, a feeling of particularly acute ***irony*** for the progression ***inside*** Q. On one hand, the $_{z}Q$ arithmetic presents some of the most astounding arithmetical *'ideo-phenomena'* we had ever encountered, as glossed above. On the other, because it models especially the ***second*** «***species***» in the «***species***»-***dialectic*** ***inside*** the «***genos***» category of "***opposition***" --

complementary opposition $\rightarrow$ ***annihilatory opposition*** $\rightarrow$ ***supplementary opposition***

-- namely, the ***annihilatory*** kind, all of those *astounding* features "go to waste" for most '***meta-modeling***' uses. That is, assigning the «***arché***» ontological category of a ***dialectical categorial progression*** to either q_{-1} or q_{+1}, in a Seldon Function, generates two equivalent progressions, one in which all of the generic **q**ualifiers in the generic progression have positive signs, the other in which all of the generic **q**ualifiers have negative signs. One thus might as well stay with $_{w}Q$ for model building, as using $_{z}Q$ in this way offers no enrichment over $_{w}Q$ modeling. Combining both «***arché***», as --

$$|\text{-}|\text{-}|_h = [q_{-1} + q_{+1}]^{2^h} = [q_{\pm 0}]^{2^h} = q_{\pm 0}$$

-- in the generic Seldon Function produces something even worse: the value of the Seldon Function for all epoc**h**s, h, is the same, namely $q_{\pm 0}$, signifying a total '''***de***-manifestation''' of *all* ontology for *all* time. This yields only '''nihilist''' 'meta-models' of the universe, and of its sub-universes, for which we have little use. That's where J2Y's new, alternative version of $_{z}Q$, which he notates by $_{z*}Q$, may come in. Its 'contra-axiomatization' of $\vdash . q_{*+z} + q_{*-z} \neq q_{*0}$, in place of our $\vdash . q_{+z} + q_{-z} = q_{\pm 0}$, may avert the "mutually annihilatory" propensity of our q_z in his q_{*z}, making the later more suitable for the formulation of more useful ***dialectical*** '***meta-models***'. We are investigating this possibility, with J2Y, right now.

Background for ***E.D. Brief #7***. **F.*E.D***. presents the system*s*-progression of the '***Gödelian Dialectic***' of the axioms-systems of the standard arithmetics, in their first-order-and-higher-logics' axiomatizations, in accord with a Dyadic Seldon Function '***meta-model***', which describes -- ideographically, and "purely-***qualitatively***" -- a '***Meta-Systematic Dialectical***' ***order-of-presentation***, and ***dialectical method-of-presentation***, of those successive systems of arithmetic. Using the notational convention that, if **X** denotes the standard number-space, or number-set, of a given kind of standard number, then that symbol, with a single underscore, **X̲**, will be used to denote its first-*and-higher-order*-logic axiomatization, we have that this **F.*E.D***. ***order-of-presentation*** can be expressed as follows, using **#** as a tag for the total «***genos***» of the standard arithmetics, comprehending all of its «***species***», in the following, progressive ordering --

$$\underline{N}_{\#} \rightarrow \underline{W}^{\#} \rightarrow \underline{Z}^{\#} \rightarrow \underline{Q}^{\#} \rightarrow \underline{R}^{\#} \rightarrow \underline{C}^{\#} \rightarrow \dots$$

-- and the Dyadic Seldon Function-based '***dialectical meta-model***' which generates that progression is --

$$\underline{)\!H\!(}_{s_{\#}}\uparrow \;=\; \left(\underline{N}_{\#}\right)^{2^{s_{\#}}\uparrow}.$$

Connected with the above-rendered ***order-of-presentation***, **F.*E.D***. presents the ***dialectical progression*** of the particular «***species***» of *first*-order-logic-*only* axiomatized ***dialectical arithmetics*** [denoted generically by **X̲**], that reside '''inside''' the «***genos***» of **F.*E.D***.'s **Q̲** '***First Dialectical Arithmetics*** meta-system', in a corresponding order --

$${}_{N}\underline{Q}_{\#} \rightarrow {}_{W}\underline{Q}^{\#} \rightarrow {}_{Z}\underline{Q}^{\#} \rightarrow {}_{Q}\underline{Q}^{\#} \rightarrow {}_{R}\underline{Q}^{\#} \rightarrow {}_{C}\underline{Q}^{\#} \rightarrow \dots$$

-- using **#** as a tag for the total «***genos***» of the **F.*E.D***. ***non***-standard, ***Dialectical Arithmetics***. The Dyadic Seldon Function-based '***dialectical meta-model***' which generates that progression is --

$$\underline{)\!H\!(}_{s_{\#}}\uparrow \;=\; \left({}_{N}\underline{Q}_{\#}\right)^{2^{s_{\#}}\uparrow}.$$

In ***E.D. Brief #5***, J2Y gave you his able **&** novel derivation of the ${}_{N}\underline{Q}$, basing the ***first stage*** of the ***dialectic*** within **Q̲**!
In ***E.D. Brief #6***, he provided his innovative derivation of the ${}_{W}\underline{Q}$, basing the ***second stage*** of the ***dialectic*** inside **Q̲**!!
In ***E.D. Brief #7***, he now presents for you a pathway to the ${}_{Z}\underline{Q}$, basing the ***third stage*** of the ***dialectic*** of the **Q̲**!!!
What J2Y has done is to illuminate a first **3** steps of the ***vast E.D***. '***double-dialectic***' / '***bi-directional dialectic***' --

$$\underline{N}_{\#} \leftrightarrow \underline{N}_{\#} \oplus {}_{N}\underline{Q}^{\#} \rightarrow \underline{N}_{\#} \oplus {}_{N}\underline{Q}^{\#} \oplus {}_{N}\underline{U}^{\#} \oplus {}_{N}\underline{M}^{\#} \rightarrow \underline{N}_{\#} \oplus {}_{N}\underline{Q}^{\#} \oplus {}_{N}\underline{U}^{\#} \oplus {}_{N}\underline{M}^{\#} \oplus {}_{N}\underline{q}_{MN}^{\#} \oplus \dots \rightarrow \dots$$

$$\Downarrow$$

$$\underline{W}_{\#} \leftrightarrow \underline{W}_{\#} \oplus {}_{W}\underline{Q}^{\#} \rightarrow \underline{W}_{\#} \oplus {}_{W}\underline{Q}^{\#} \oplus {}_{W}\underline{U}^{\#} \oplus {}_{W}\underline{M}^{\#} \rightarrow \underline{W}_{\#} \oplus {}_{W}\underline{Q}^{\#} \oplus {}_{W}\underline{U}^{\#} \oplus {}_{W}\underline{M}^{\#} \oplus {}_{W}\underline{q}_{MW}^{\#} \oplus \dots \rightarrow \dots$$

$$\Downarrow$$

$$\underline{Z}_{\#} \leftrightarrow \underline{Z}_{\#} \oplus {}_{Z}\underline{Q}^{\#} \rightarrow \underline{Z}_{\#} \oplus {}_{Z}\underline{Q}^{\#} \oplus {}_{Z}\underline{U}^{\#} \oplus {}_{Z}\underline{M}^{\#} \rightarrow \underline{Z}_{\#} \oplus {}_{Z}\underline{Q}^{\#} \oplus {}_{Z}\underline{U}^{\#} \oplus {}_{Z}\underline{M}^{\#} \oplus {}_{Z}\underline{q}_{MZ}^{\#} \oplus \dots \rightarrow \dots$$

$$\Downarrow$$

$$\vdots \qquad \dots \rightarrow \dots$$

The axioms of *the core axioms sub-set* of the **F.*E*.*D*.** $_{\mathbf{z}}\underline{\mathbf{Q}}$ axioms-system for ***dialectical arithmetic*** are as follows --

(§1) $[\forall z \in \mathbf{Z}][\,\underline{q}_z \in {}_{z}\underline{Q}\,]$ [the axiom of «***aufheben***» *connexion*, or of *subsumption* [of the *subsumption* of the **Z** by the $_{z}\underline{Q}$]].

(§2) $[\forall z \in \mathbf{Z}][[\,\underline{q}_z \in {}_{z}\underline{Q}\,] \;\Rightarrow\; [\,\underline{s}\underline{q}_z = \underline{q}_{z+1} \in {}_{z}\underline{Q}\,]\,]$ [axiom of inclusion of $_{z}\underline{Q}$ **q**ualifiers' ontological successors].

(§3) $[\forall j, k \in \mathbf{Z}][\,[\,[[\underline{q}_j, \underline{q}_k \in {}_{z}\underline{Q}] \;\&\; [\underline{q}_j \neq \underline{q}_k]] \;\Rightarrow\; [\,\underline{s}\underline{q}_j \neq \underline{s}\underline{q}_k\,]\,]\,]$ [axiom of $_{z}\underline{Q}$ successor ***uniqueness***].

(§4) $[\forall j, k \in \mathbf{Z}][\,[\,j \gtrless k\,] \;\Rightarrow\; [\,\underline{q}_j \neq \underline{q}_k\,]\,]$ [axiom of the ***qualitative uniqueness*** of distinct **Z**-based ontological ***qualifiers***].

(§5) $[\forall z \in \mathbf{Z}][\,\underline{q}_z + \underline{q}_z \;=\; \underline{q}_z\,]$ [axiom of $_{z}\underline{Q}$ ***idempotent addition***; of ***ontological category*** [ontological **q**ualifier] '***unquantifiability***'].

(§6) $[\forall\, i, j, k \in \mathbf{Z} - \{\pm 0\}][\,[\,j \gtrless \pm k\,] \;\Rightarrow\; [\,\underline{q}_j \pm \underline{q}_k \neq \underline{q}_i\,]\,]$ [axiom of ***irreducibility*** for **Z**-based ***qualitative sums***].

(§7) $[\forall\, j, k \in \mathbf{Z}][\,\underline{q}_j \times \underline{q}_k \;=\; \underline{q}_j + \underline{q}_k + \underline{q}_{j+k}\,]$ [axiom of '***the meta-genealogical evolute product rule***' for $_{z}\underline{Q}$ **q**ualifier $\times$].

(§8) $[\forall\, j, k \in \mathbf{Z}][\,\underline{q}_j + \underline{q}_k \;=\; \underline{q}_j + \underline{q}_k\,]$ [axiom of ***+commutativity*** of **Z**-based ***qualitative*** / ***qualifier sums***].

(§9) $[\forall\, z \in \mathbf{Z}][\,\underline{q}_z + q_{\pm 0} \;=\; q_{\pm 0} + \underline{q}_z \;=\; \underline{q}_z\,]$ [axiom of the ***+identity element*** for the $_{z}\underline{Q}$].

(§10) $[\forall\, z \in \mathbf{Z}][\,\underline{q}_z + [-\underline{q}_{+z}]^{+1} \equiv [\,\underline{q}_{+z}/q_{\pm 0}] + [\,q_{\pm 0}/\underline{q}_{+z}\,] \;=\; \underline{q}_{+z} + \underline{q}_{-z} \;=\; q_{\pm 0}\,]$ [axiom of the ***+inverse elements*** in $_{z}\underline{Q}$].

(§11) $[\forall\, z \in \mathbf{Z}][\,\exists[-\underline{q}_{+z}]^{+1} \equiv +\underline{q}_{-z}^{\;+1} \equiv +\underline{q}_{+z}^{\;-1} \equiv q_{\pm 0}/\underline{q}_{+z} \,|\, \underline{q}_{+z} \times \underline{q}_{-z} \;=\; q_{\pm 0}\,]$ [axiom of the ***×inverse elements*** in $_{z}\underline{Q}$].

-- wherein **s** denotes the "Peano **s**uccessor operator", $\mathbf{s}(z) \;=\; z + 1$, and wherein $\underline{\mathbf{s}}$ denotes the $_{z}\underline{Q}$ version of that **s**uccessor function, $\underline{\mathbf{s}}[\,\underline{q}_z\,] \;=\; \underline{q}_{s(z)} \;=\; \underline{q}_{z+1}$.

Each successor-system in the '***Gödelian Dialectic***' of the **F.*E*.*D*.** axioms-systems progression --

$_{N}\underline{Q}_{\#} \;\ni\; {}_{W}\underline{Q}^{\#} \;\ni\; {}_{Z}\underline{Q}^{\#} \;\ldots$

is *more complex*, more '''[*thought*-]*concrete*''', and more "*definite*" -- richer in "*determinations*", in "*features*", in '***ideo***-*ontology*' -- than are its predecessor-systems, and hence is also richer in ideographical-linguistic expressive and descriptive capabilities and facilities than they are. Each successor-system also «***aufheben***»-'"***contains***"', «***aufheben***»-'"***elevates***"', and «***aufheben***»-***transforms***/-'"***negates***"' ***all*** of its predecessor-systems, and constitutes a '"conservative extension"' of its *immediate* predecessor-system. Corresponding to the first three **s**tages of **F.*E*.*D*.**'s ***dialectical presentation*** of the progression ***within*** $\underline{\mathbf{Q}}$, expressed above, is that, to **s**tage $\mathbf{s}_{\#} \;=\; \mathbf{3}$, of **F.*E*.*D*.**'s ***dialectical presentation*** of the standard systems of arithmetic:

$\underline{)\text{H}(}_{s_{\#}=0} \;=\; (\!(\,\underline{N}_{\#}\,)\!)^{2^{0}} \;=\; \underline{N}_{\#}$; with '$\vdash\!\oplus\!\dashv$' signing '*antagonistic addition*'/'*summings of **opposite** **q**ualities*' --

$\underline{)\text{H}(}_{s_{\#}=1} \;=\; (\!(\,\underline{N}_{\#}\,)\!)^{2^{1}} \;=\; \underline{N}_{\#} \vdash\!\oplus\!\dashv \underline{A}^{\#}$, wherein **A** denotes the "**A**ught"-numbers, $\mathbf{A} \equiv \{\,[\forall n \in \mathbf{N}][\,n - n\,]\,\}$;

$\underline{)\text{H}(}_{s_{\#}=2} \;=\; (\!(\,\underline{N}_{\#}\,)\!)^{2^{2}} \;=\; \underline{N}_{\#} \oplus \underline{A}^{\#} \oplus \underline{q}_{AN}^{\#} \vdash\!\oplus\!\dashv \underline{M}^{\#} \equiv \underline{W}^{\#} \vdash\!\oplus\!\dashv \underline{M}^{\#}$; $\underline{q}_{AN}^{\#}$ unifying $\underline{A}^{\#}$ & $\underline{N}_{\#}$;

$\mathbf{M} \equiv$ the "**M**inus" numbers $\equiv \{\,[\forall n > 1 \in \mathbf{N}][\,1 - n\,]\,\}$;

$\underline{)\text{H}(}_{s_{\#}=3} \;=\; (\!(\,\underline{N}_{\#}\,)\!)^{2^{3}} \;=\; \underline{N}_{\#} \oplus \underline{A}^{\#} \oplus \underline{q}_{AN}^{\#} \oplus \underline{M}^{\#} \oplus \underline{q}_{MN}^{\#} \oplus \underline{q}_{MA}^{\#} \oplus \underline{q}_{MAN}^{\#} \vdash\!\oplus\!\dashv \underline{F}^{\#} \;\equiv$

$\underline{Z}^{\#} \vdash\!\oplus\!\dashv \underline{F}^{\#}$, wherein $\underline{q}_{MN}^{\#} \oplus \underline{q}_{MA}^{\#} \oplus \underline{q}_{MAN}^{\#}$ reconcile $\underline{M}^{\#}$ with $\underline{N}_{\#} \oplus \underline{A}^{\#} \oplus \underline{q}_{AN}^{\#}$, & with $\mathbf{F} \equiv$ the **F**ractional Numbers,

$\{\,[\forall z_j < z_k \neq \pm 0 \in \mathbf{Z}][\,z_j/z_k\,]\,\}$.