F.*E*.*D*. Preface to *E*.*D*. *Brief* #5, by Guest Author "J2Y"

by *Hermes de Nemores*, General Secretary to the F.*E*.*D*. General Council

Summary. Our new guest author, known pseudonymously as "Joy-to-You", and whom I shall reference, herein, using the nickname with which he often references himself in our correspondence -- "J2Y" -- has provided to you, our readers, *a new, short* [just **7** pages], *and highly-accessible* «***entrée***» into the F.*E*.*D*. '***First Dialectical Arithmetic***', the $_{N}Q$ system of ***dialectical arithmetic***, with its *core* set, or space, of ***dialectical***, '***meta-Natural meta-numbers***' --

$$_{N}Q \equiv \{ q_1, q_2, q_3, \ldots \}.$$

What J2Y has accomplished for you is to develop a *single* new '''*idea-object*''', denoted C_N, with which he shows how to *co-generate*, in a coordinated way, three of the four key axioms that distinguish the $_{N}Q$ axioms-system from that of the Standard **N**atural Numbers[, the numbers of the set $N \equiv \{ 1, 2, 3, \ldots \}$], and thus how to ***unify***, the *core* characteristics of the $_{N}Q$ system that those three axioms codify.

Background. We had long treasured the operational *"open-ness"* of the $_{N}Q$ space -- its [*possibly unique*] feature that the mutual addition, or mutual multiplication, of any two distinct '***meta-number***' values, taken from *"inside"* the $_{N}Q$ space, yields a value which resides *"outside"* of that space.

Most of the "abstract algebras"[, or 'abstract arithmetics',] which modern mathematics treats are *operationally "closed"* -- any of their operations between any pair of their values produces a value which is also a member of their set or space.

The $_{N}Q$ arithmetic-system, on the contrary, is completely "open" in its *'intra-operations'*, i.e., when those operations are engaged between *distinct* elements of $_{N}Q$, for its ontological-categories' *addition operation*, and between *any* elements -- meaning including between non-distinct elements -- for its *ontological multiplication operation*.

If the "space" of the elements of the $_{N}Q$ set is 'geometrized' as a space of '***dialectors***' -- if each distinct element of $_{N}Q$ is imagined as the embodiment of a unique spatial *direction* and '''*dimension*''', represented as an *indivisible, unit-length, oriented* line-segment, mutually orthogonal with each of those representing every other such element, and equipped with a *non*-vector ontological multiplication rule, then the addition of any *distinct* pair, or the multiplication of *any* pair, of *such* '***meta-vectors***', or '***dialectors***', yields a $2^{1/2}$-unit(s)-length, ***diagonal*** '***dialector***', incommensurable with its unit-length summands or factors.

The $_{N}Q$ space is *"operationally open*[ed]" by virtue of this '''***diagonalizing***''' process, a process which we describe as the '***diagonal self-transcendence***' of the $_{N}Q$ space.

Its multiplicative sub-process is key to the capability of the $_{N}Q$ algebra to express '***dynamical ontologies***' or '***ontology-dynamics***' -- to "model" what we term '*processes of* ***onto-dynamasis***'.

However, we had not heretofore explored the "larger" space -- the space that contains the $_{N}Q$ space, but which also "exceeds" it -- containing [also] all possible sums[, and all possible products, whose results are also among those sums,] of the "singleton" **q**ualifiers contained in $_{N}Q$, and, thereby, constituting the '''closure''' -- the '''closure space"', or the '''closure set''' -- for $_{N}Q$ and for the [its] $_{N}Q$.

Such an exploratory expedition is precisely what J2Y recounts in his new essay, posted here as ***E*.*D*. *Brief* #5** !

Overview. The axioms of *the core axioms sub-set* of the $_{\mathbf{N}}\mathbf{Q}$ axioms-system for ***dialectical arithmetic*** are as follows --

(§1) $\mathbf{q}_1 \in {}_{\mathbf{N}}\mathbf{Q}$ [the axiom of «***arché***» inclusion].

(§2) $[\forall n \in \mathbf{N}][[\mathbf{q}_n \in {}_{\mathbf{N}}\mathbf{Q}] \Rightarrow [\mathbf{s}\mathbf{q}_n = \mathbf{q}_{n+1} \in {}_{\mathbf{N}}\mathbf{Q}]]$ [the axiom of inclusion of ontological successors].

(§3) $[\forall j, k \in \mathbf{N}][\,[\,[[\mathbf{q}_j, \mathbf{q}_k \in {}_{\mathbf{N}}\mathbf{Q}] \,\&\, [\mathbf{q}_j \neq \mathbf{q}_k]] \Rightarrow [\mathbf{s}\mathbf{q}_j \neq \mathbf{s}\mathbf{q}_k]\,]]$ [the axiom of categorial distinctness].

(§4) $[\forall x \in \mathbf{N}][\,\neg[\exists \mathbf{q}_x \in {}_{\mathbf{N}}\mathbf{Q}] \mid [\mathbf{s}\mathbf{q}_x = \mathbf{q}_1]\,]$ [the axiom of the '***archéonicity***' of the «***arché***»].

(§5) $[\forall n \in \mathbf{N}][\mathbf{q}_n \in {}_{\mathbf{N}}\mathbf{Q}]$ [the axiom of «***aufheben***» connexion, and of subsumption [of the subsumption of the $\mathbf{N}$ by the $_{\mathbf{N}}\mathbf{Q}$]].

(§6) $[\forall j, k \in \mathbf{N}][\,[j \gtrless k] \Rightarrow [\mathbf{q}_j \not\gtrless \mathbf{q}_k]\,]$ [the axiom of the ***qualitative inequality*** of distinct ontological ***qualifiers***].

(§7) $[\forall n \in \mathbf{N}][\mathbf{q}_n + \mathbf{q}_n = \mathbf{q}_n]$ [the axiom of the *idempotent addition* / of ***ontological category*** [ontological **q**ualifier] ***uniqueness***].

(§8) $[\forall\, i, j, k \in \mathbf{N}][\,[j \gtrless k] \Rightarrow [\mathbf{q}_j + \mathbf{q}_k \not\gtrless \mathbf{q}_i]\,]$ [the axiom of the ***irreducibility*** of ontological ***qualitative differences***].

(§9) $[\forall\, j, k \in \mathbf{N}][\mathbf{q}_j \times \mathbf{q}_k = \mathbf{q}_k + \mathbf{q}_{k+j}]$ [the axiom of *the* ***double-«aufheben»*** *evolute product rule* for ontological multiplication].

-- wherein $\mathbf{s}$ denotes the "Peano **s**uccessor operator", $\mathbf{s(n)} = \mathbf{n+1}$, and wherein $\mathbf{s}$ denotes the $_{\mathbf{N}}\mathbf{Q}$ version of that **s**uccessor function, $\mathbf{s}[\mathbf{q}_n] = \mathbf{q}_{s(n)} = \mathbf{q}_{n+1}$.

The first four of these nine core axioms are just the $_{\mathbf{N}}\mathbf{Q}$ versions of the four, first order "Peano Postulates" for the "Standard **N**atural Numbers". They do their part in demonstrating that the $_{\mathbf{N}}\mathbf{Q}$ arithmetic is one of the "***Non***-Standard Models" of the "**N**atural Numbers", whose *ineluctable* '''*co-inherence*''', together with the "Standard Model", in the first order Peano Postulates, was predicted as the conjoint implication of two of the deepest theorems in modern mathematics. Axiom five shows how the "Standard Arithmetic" of the $\mathbf{N}$, and the "***Non***-Standard-**N**atural Arithmetic" of the $_{\mathbf{N}}\mathbf{Q}$, tie together, with the $\mathbf{N}$ subsumed by the $_{\mathbf{N}}\mathbf{Q}$. Axiom six helps to define the meaning of the relation of '***qualitative inequality***', denoted by the new sign '$\not\gtrless$', which holds between any *distinct* pair of $_{\mathbf{N}}\mathbf{Q}$ '***meta-numbers***', and to describe how that relation relates to the standard relation of ***quantitative inequality*** among *distinct* subscripts, $\mathbf{n}$, of the $\{\,\mathbf{q}_n\,\}$.

It is axioms seven, eight, and nine that describe the core characteristics that most constitute the '***non****-standard-ness*' of the $_{\mathbf{N}}\mathbf{Q}$ '***Non***-Standard Model of the **N**atural Numbers". *And it is those final three axioms which* J2Y*'s approach unifies*.

He achieves this unification, first, by conceiving of the '''closure'''['''closure space'''] for all multiplication and addition operations of/by/on the $_{\mathbf{N}}\mathbf{Q}$ **q**ualifiers, a new concept he names **{ Cumula }**, or *"**Open Qualifier Space**"*.

He then constructs, on the basis of that *"**Open Qualifier Space**"* concept, a sub-set of that space, the sub-space which contains all of, and only, the '***archéonic consecua***' -- all of the ***consecutive*** *sums* of $_{\mathbf{N}}\mathbf{Q}$ **q**ualifiers that begin with the "beginner", or «***arché***», of the $_{\mathbf{N}}\mathbf{Q}$, namely $\mathbf{q}_1$ -- i.e., all of the $_{\mathbf{N}}\mathbf{Q}$ **q**ualifier '***cumula***' that are generated by, e.g., Dyadic or Triadic '''Seldon Functions''' for consecutive successive values, starting with $\mathbf{0}$, of their $\boldsymbol{\tau}$ [epoch] or $\mathbf{s}$ [presentation-**s**tep] parameters, and which he calls "**N**-***Cumulation Space***", denoted by $\mathbf{C_N}$.

The crescendo ensues by applying a **q**ualifier ***finite*** '''***∂ifference operator***''', $\partial(\,\_\,)$, defined within the $\mathbf{C_N}$ sub-space, to "*∂erive*" $_{\mathbf{N}}\mathbf{Q}$ from $\mathbf{C_N}$, and, by considering implications of the connexion(s) between $\mathbf{C_N}$ and $_{\mathbf{N}}\mathbf{Q}$, to derive / unify axioms seven, eight, and nine. Regarding axiom **§8**, of '***cumula irreducibility***', J2Y's work actually implies a corollary --

$$[\,[\forall\, i, j, k \in \mathbf{N}][\,[j \gtrless k] \Rightarrow [\mathbf{q}_j + \mathbf{q}_k \not\gtrless \mathbf{q}_i]\,]\,] \Rightarrow [\,[\mathbf{q}_j + \mathbf{q}_k] \notin {}_{\mathbf{N}}\mathbf{Q}]$$

-- since $_{\mathbf{N}}\mathbf{Q}$ contains only '''singleton''', or '***recti-dialector***', unit-length **q**ualifiers, and not any '***diagonal dialector***' **q**ualifiers, of incommensurable moduli [of length $=$ square root of $\mathbf{2}$], i.e., not any binary/dyadic *sums* of **q**ualifiers.